

Bitext parsing

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3/5/17

Context-Free Grammars

A **CFG** grammar G is denoted by

- a finite set of **nonterminal** symbols N
- a finite set of **terminal** symbols Σ with $\Sigma \cap N = \emptyset$
- a finite set R of **rules** of the form $X \rightarrow \alpha$ where
 - $X \in N$ and $\alpha \in (\Sigma \cup N)^*$
- $S \in N$ a distinguished **start** symbol

Let ε denote an **empty** string

Example CFG

$S \rightarrow NP VP$

$Vi \rightarrow \text{sleeps}$

$VP \rightarrow Vi$

$Vt \rightarrow \text{saw}$

$VP \rightarrow Vt NP$

$NN \rightarrow \text{man}$

$VP \rightarrow VP PP$

$NN \rightarrow \text{dog}$

$NP \rightarrow DT NN$

$NN \rightarrow \text{telescope}$

$NP \rightarrow NP PP$

$DT \rightarrow \text{the}$

$PP \rightarrow IN NP$

$IN \rightarrow \text{with}$

Generative Device

Left-most derivation

- sequence of strings $\mathbf{s}_1 \dots \mathbf{s}_n$
 - $\mathbf{s}_1 = S$
 - $\mathbf{s}_n \in \Sigma^*$
 - $\mathbf{s}_{i \geq 2}$ derived from \mathbf{s}_{i-1} by picking the left-most nonterminal X
 - replacing it by some a such that $X \rightarrow a \in R$

Example of Derivation

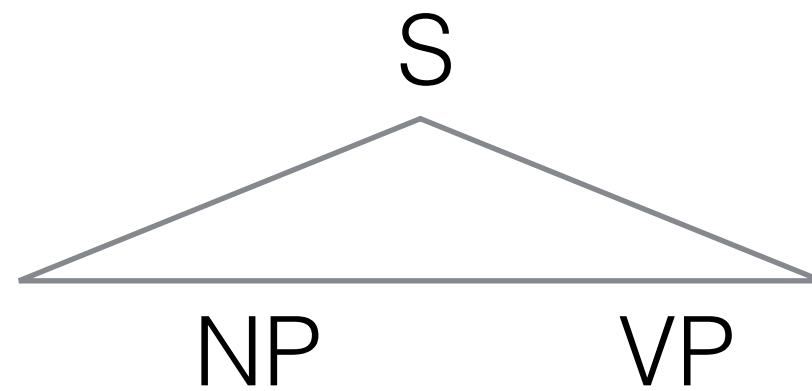
		Substitution
s ₁ =	S	S → NP VP
s ₂ =	NP VP	NP → DT NN
s ₃ =	DT NN VP	DT → the
s ₄ =	the NN VP	NN → man
s ₅ =	the man VP	VP → Vi
s ₆ =	the man Vi	Vi → sleeps
s ₇ =	the man sleeps	
s ₇ =	S ⇒* the man sleeps	

Example of Generation

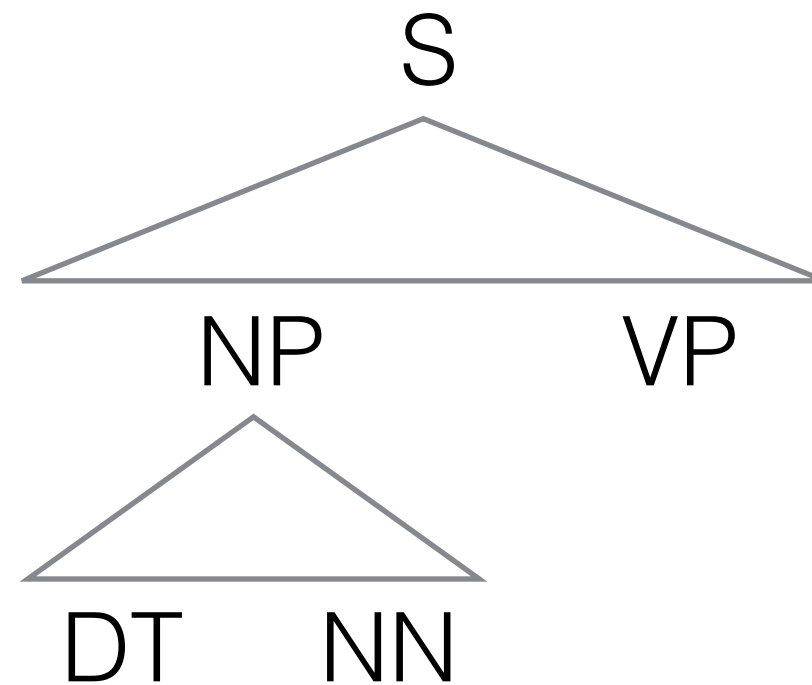
Example of Generation

S

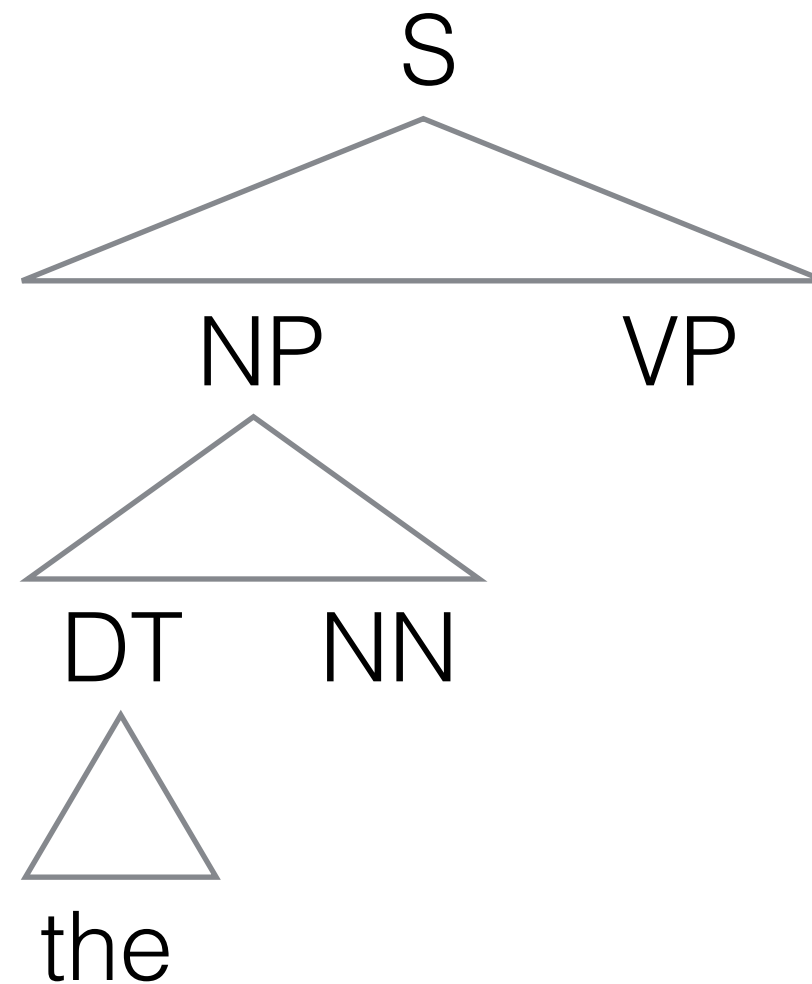
Example of Generation



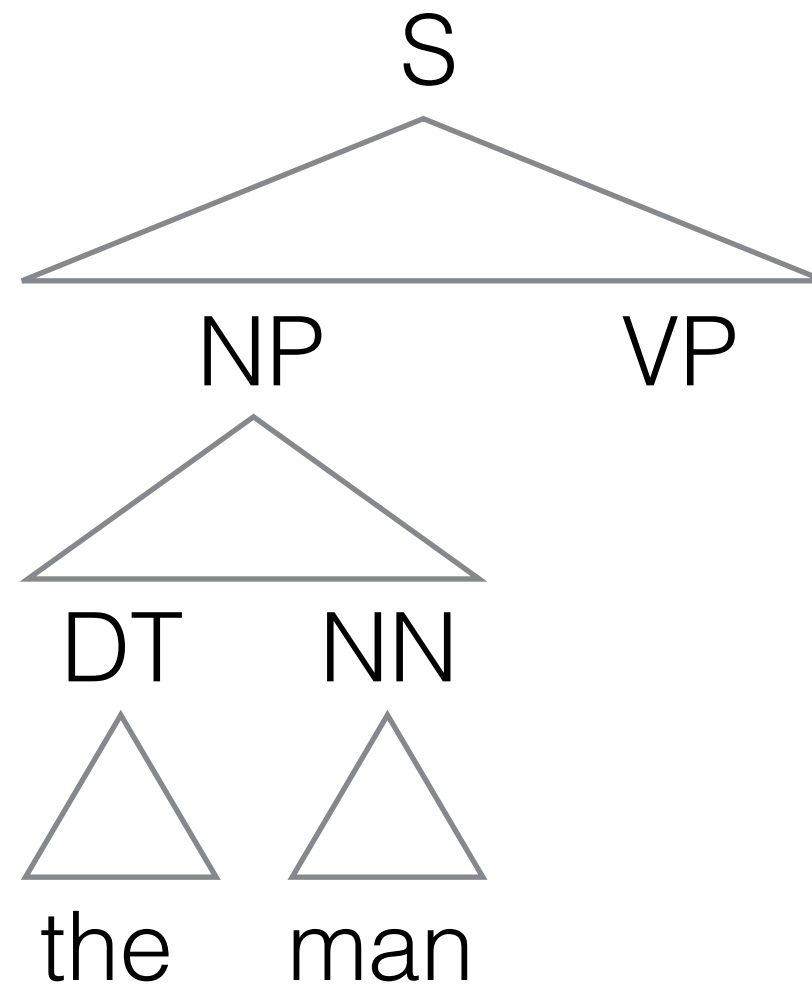
Example of Generation



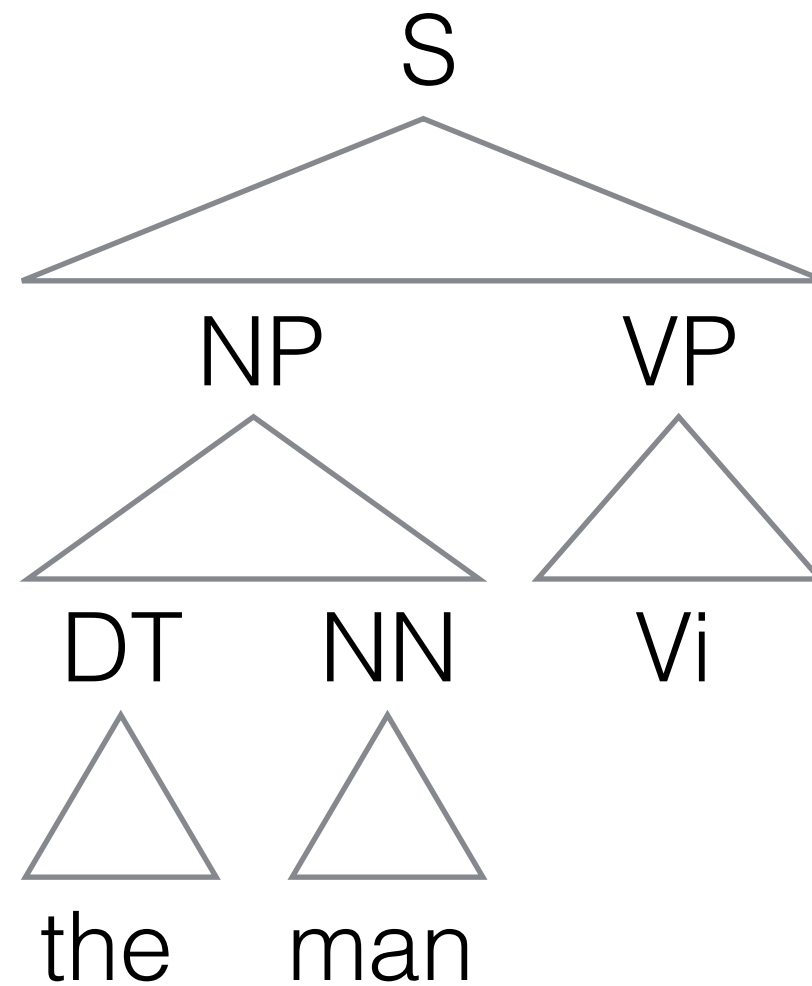
Example of Generation



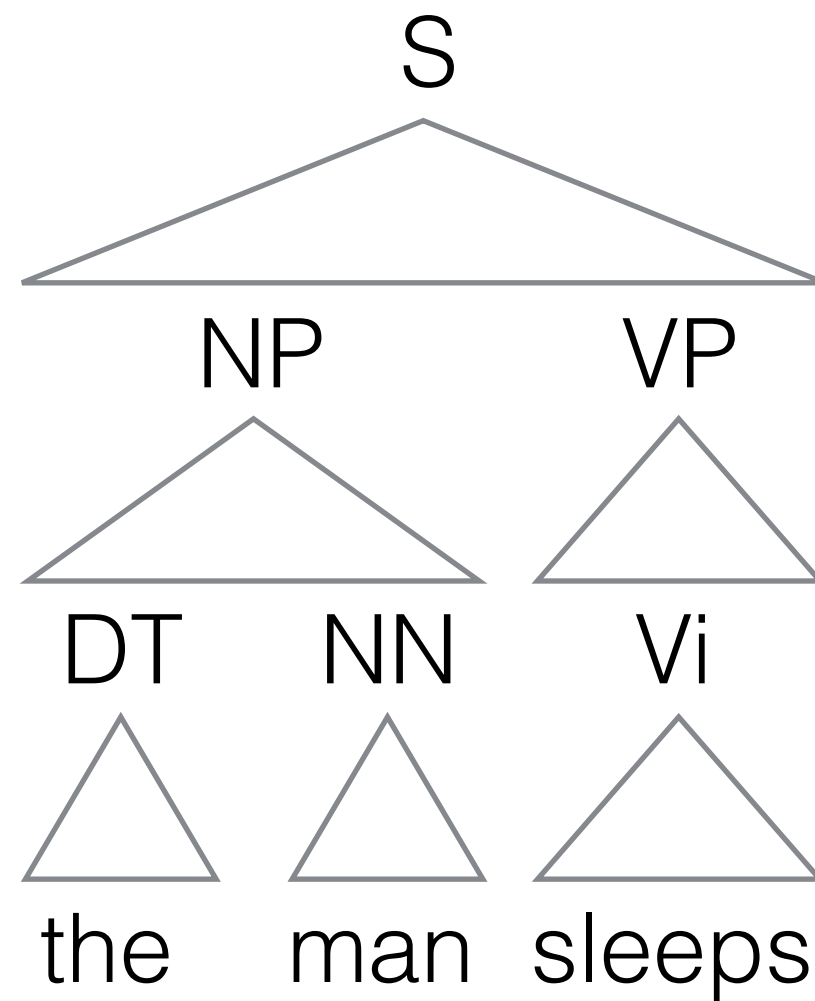
Example of Generation



Example of Generation



Example of Generation



Example of Recognition

Example of Recognition

The man saw the dog

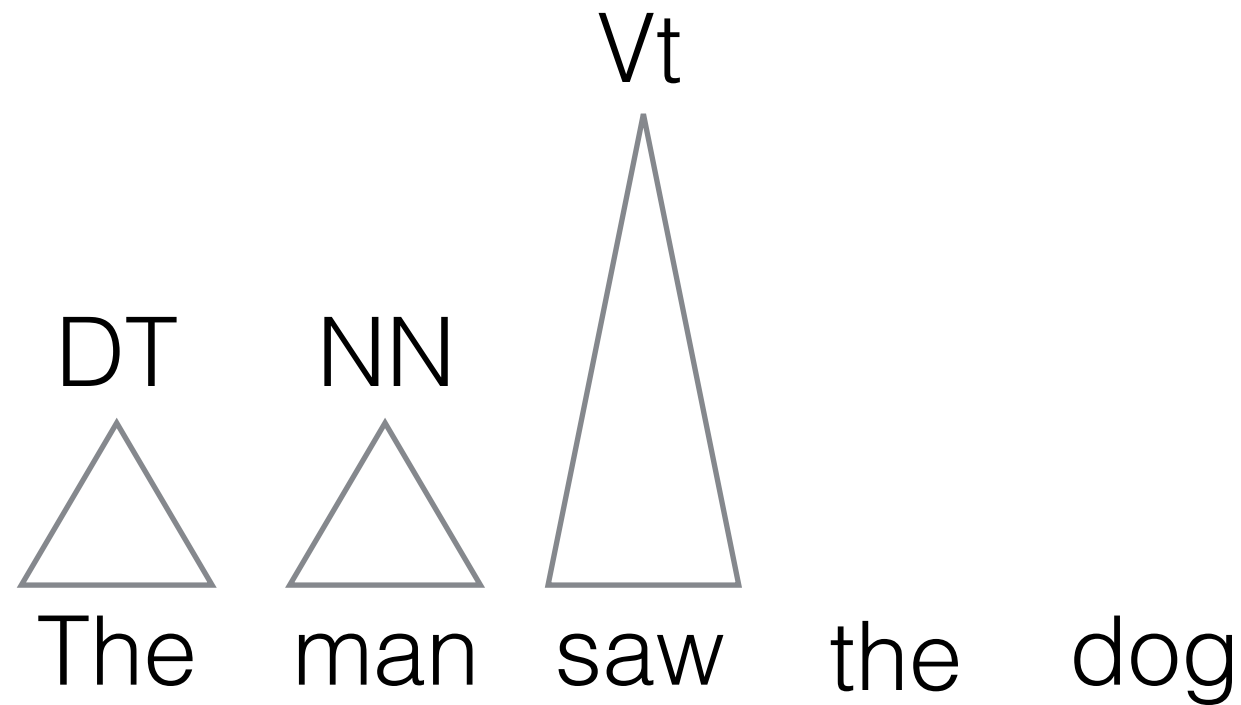
Example of Recognition

DT
△
The man saw the dog

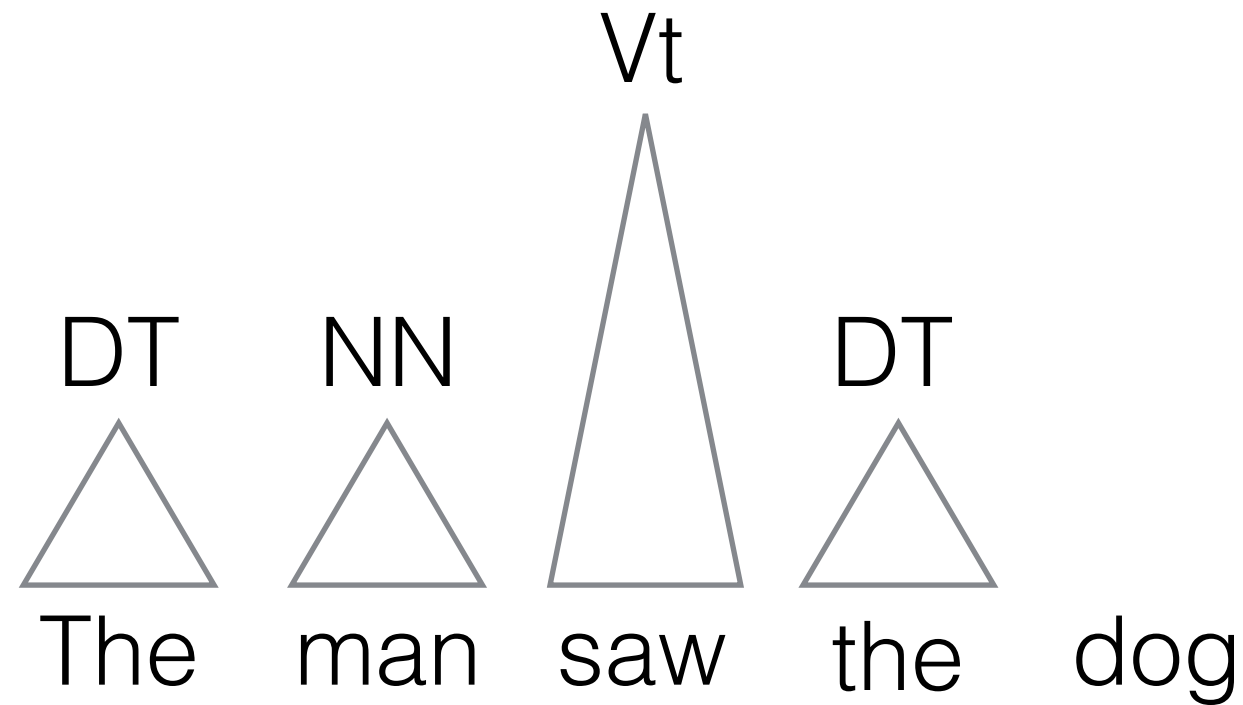
Example of Recognition

DT NN
△ △
The man saw the dog

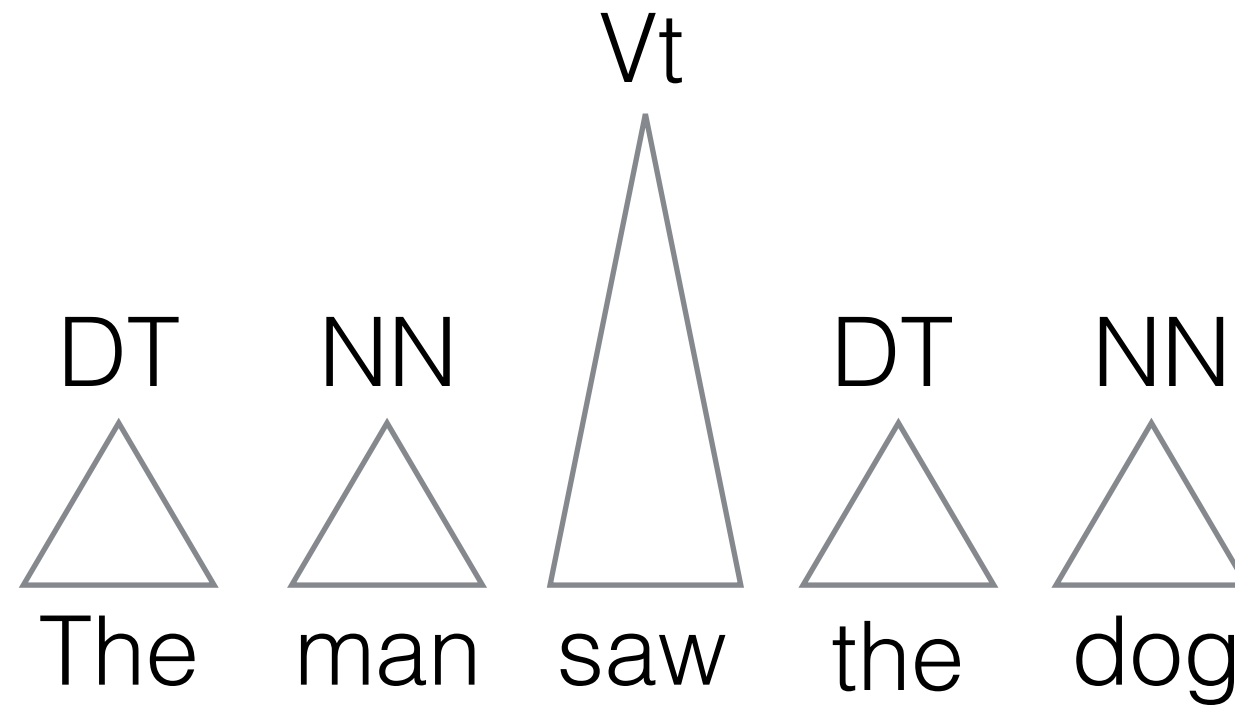
Example of Recognition



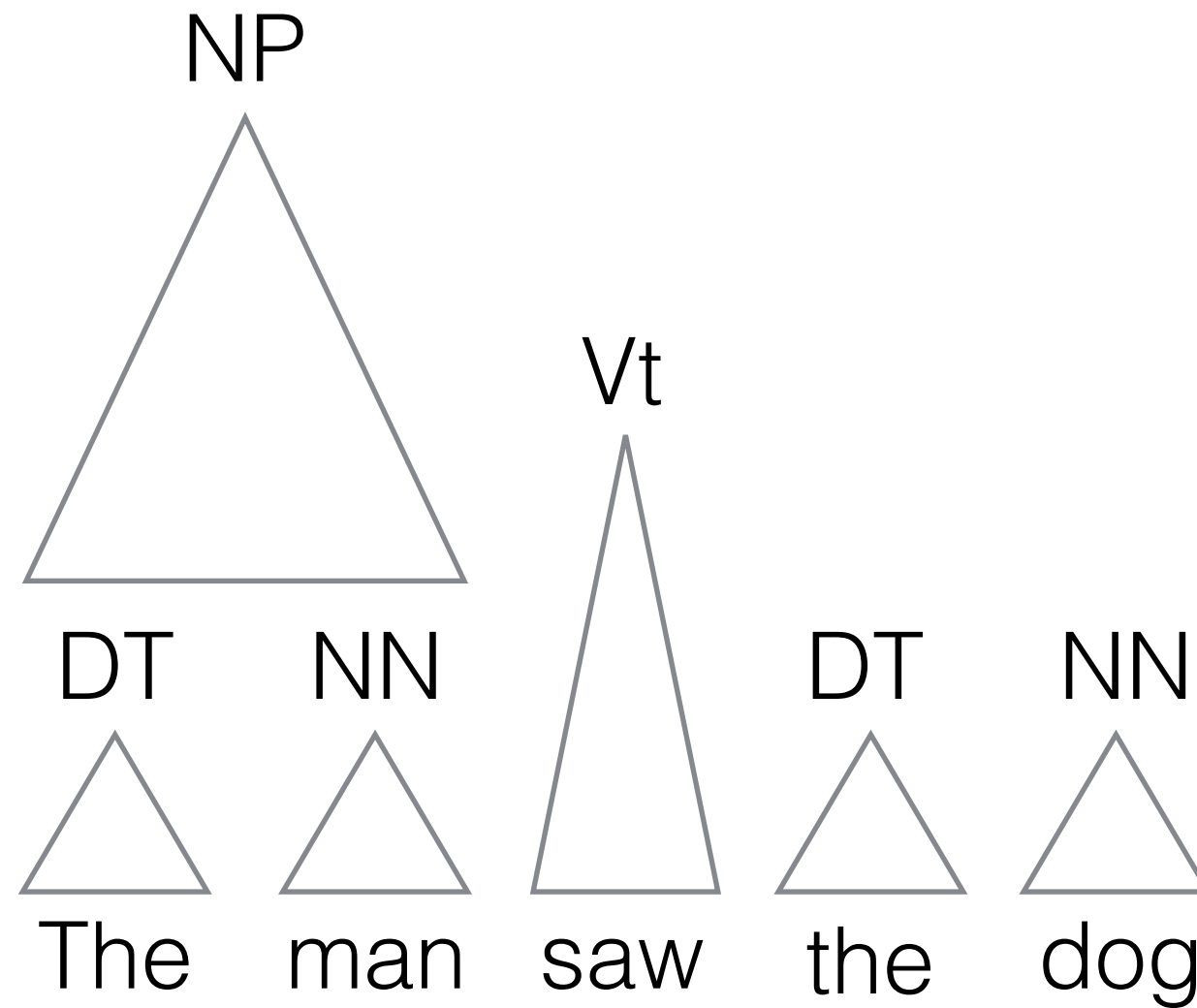
Example of Recognition



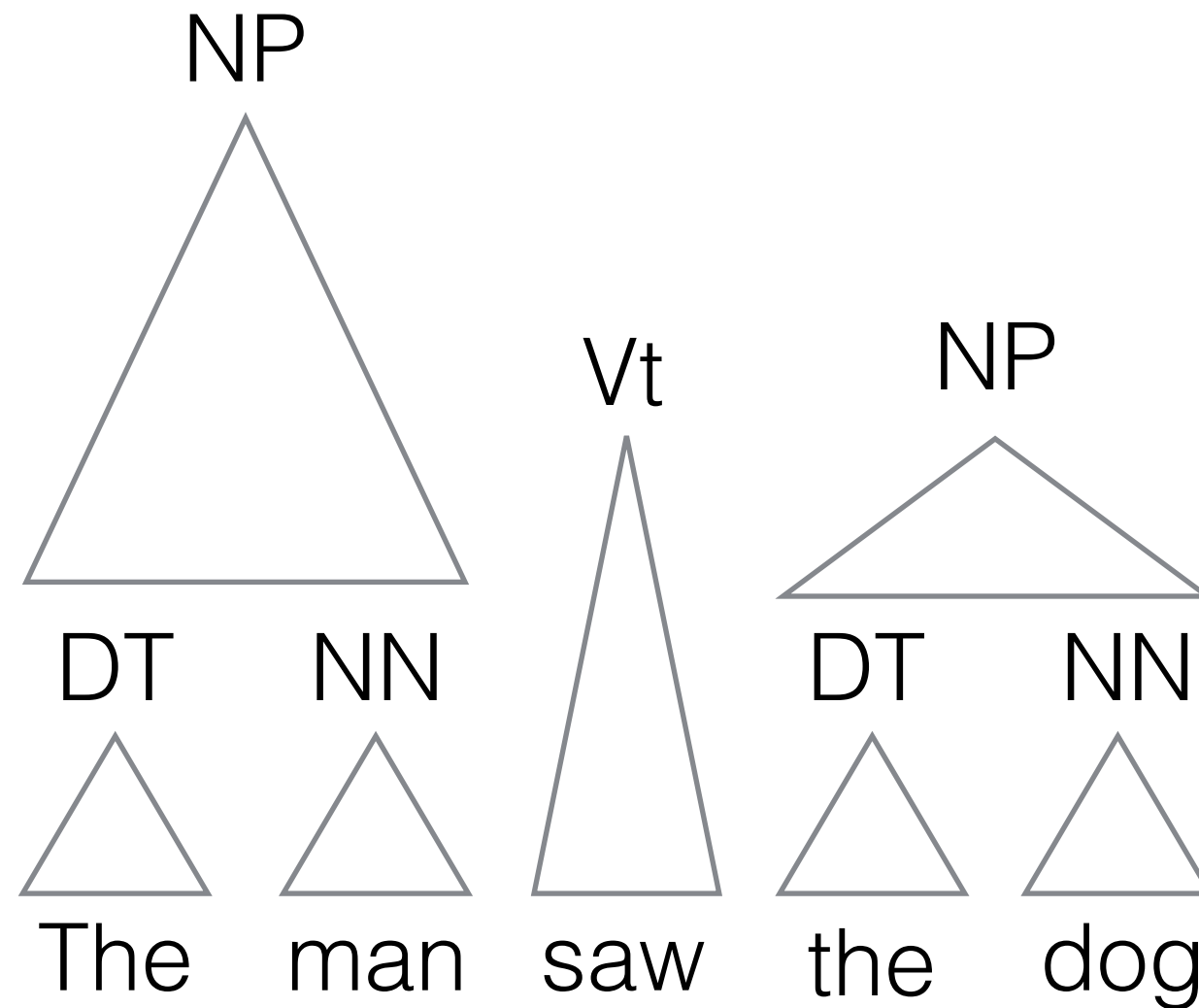
Example of Recognition



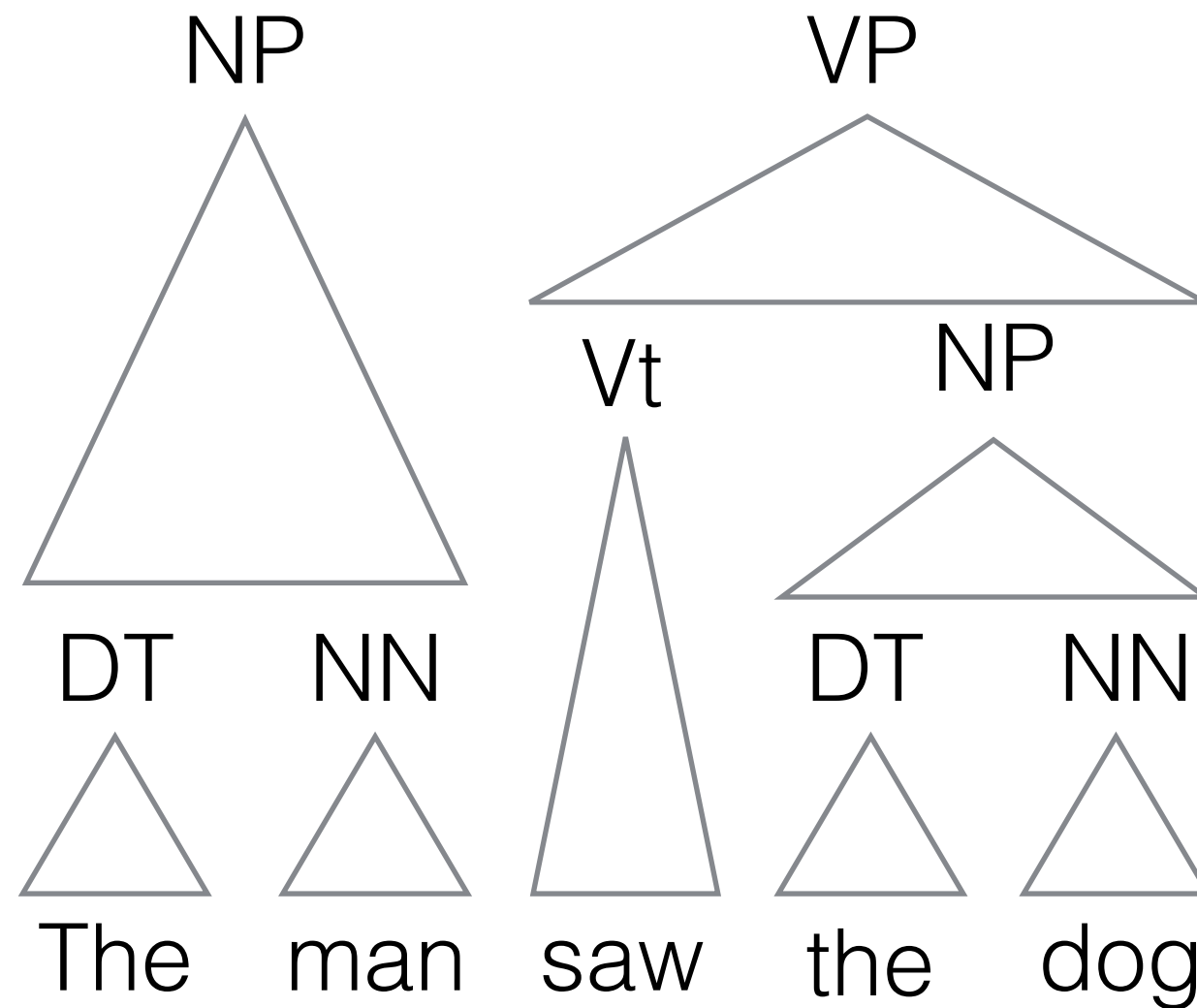
Example of Recognition



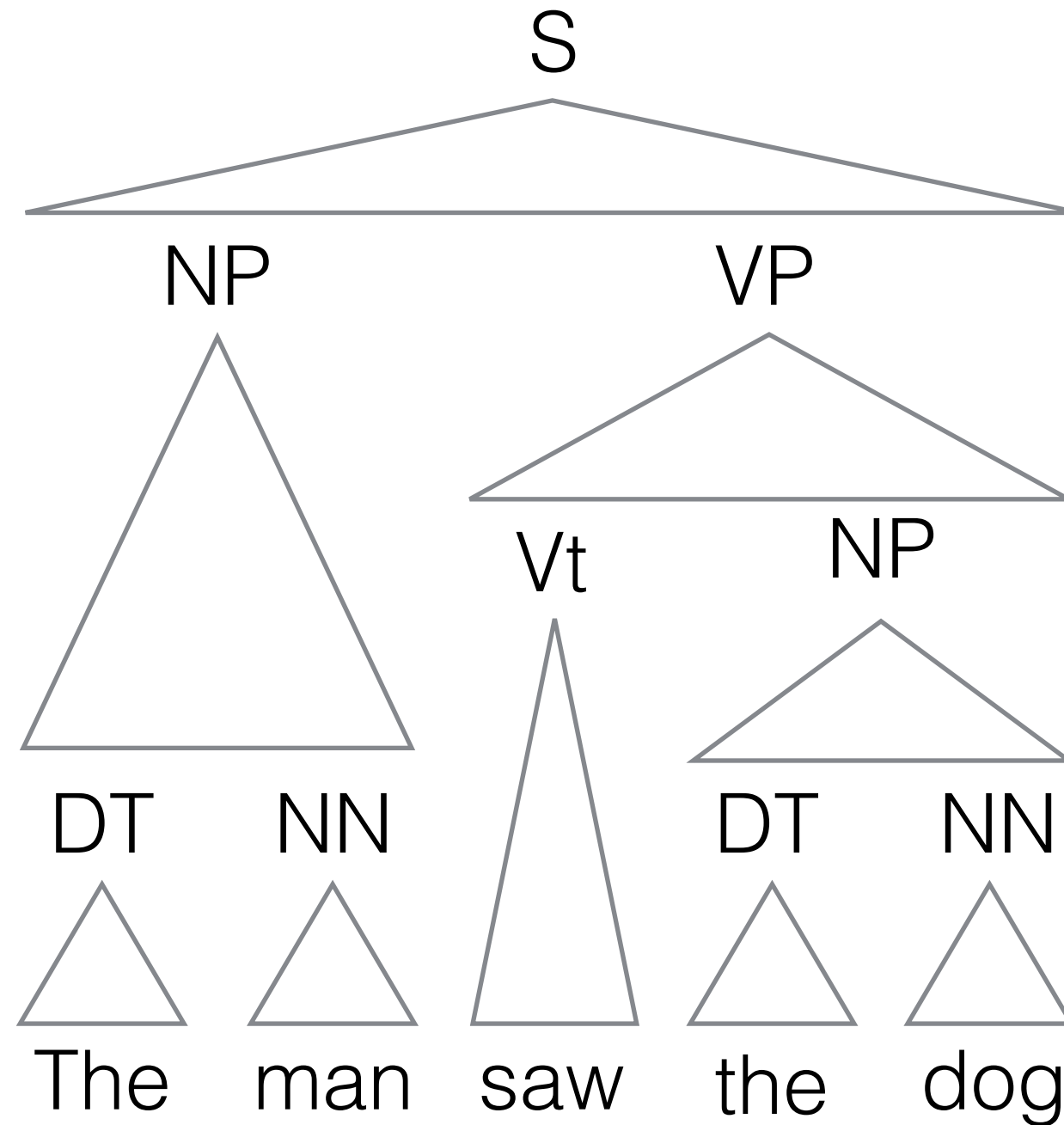
Example of Recognition



Example of Recognition



Example of Recognition



Language

A string $\mathbf{s} = s_1 \dots s_n$ is generated/accepted by G if

$$S \Rightarrow^* \mathbf{s}$$

\Rightarrow^* denotes a sequence of rule applications

Language of G

$$L(G) = \{\mathbf{s} : S \Rightarrow^* \mathbf{s}\} \subseteq \Sigma^*$$

Chomsky Normal Form

Every CFG is weakly equivalent to another such that

- $X \rightarrow YZ$ where $X, Y, Z \in N$
- $X \rightarrow w$ where $w \in \Sigma$
- and possibly $S \rightarrow \varepsilon$

[Hopcroft and Ullman, 1979]

Parsing as Deduction

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Deductive process to prove claims about grammaticality
[Shieber et al., 1995]

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- complexity determined by inspection
- dynamic program follows directly

Parsing as Deduction

Deductive process to prove claims about grammaticality
[Shieber et al., 1995]

- focus on strategy rather than implementation
- soundness/completeness easier to prove
- complexity determined by inspection
- dynamic program follows directly
- generality

Deductive systems

Item: a statement / intermediate sound result

- formula or schemata expressed with variables

Inference rule: statement derived from existing items

- $\frac{A_1 \dots A_m}{B}$ (condition) where A_i and B are items
 - A_i are called antecedents
 - B is called consequent

Deductive program

Axioms: trivial items

- do not depend on previous statements

Goal: states that a proof exists

Proof:

- start from axioms
- exhaustively deduce items
 - never twice under the same premises
- accept if goal is proven

Shift-Reduce Example

Input: *the man sleeps*

S → NP VP

VP → Vi

VP → Vt NP

VP → VP PP

NP → DT NN

NP → NP PP

PP → IN NP

Vi → sleeps

Vt → saw

NN → man

NN → dog

NN → telescope

DT → the

IN → with

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Rule	Condition	Statement	Queue
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Axiom	1	[•,0]	1

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Rule	Condition	Statement	Queue
Axiom	1	[•,0]	1
Shift: [1]	2	[the•,1]	2

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Axiom		1 [•,0]	1
Shift: [1]		2 [the•,1]	2
Reduce: [2]	DT → the	3 [DT•,1]	3

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Shift: [3]		4 [DT man •, 2]	4

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Reduce: [4]	NN → man	5 [DT NN •, 2]	5
Reduce: [5]	NP → DT NN	6 [NP •, 2]	6

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Shift: [6]		7 [NP sleeps •, 3]	7

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Shift: [6]		7 [NP sleeps •, 3]	7
Reduce: [7]	Vi → sleeps	8 [NP Vi •, 3]	8

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Reduce: [2]	DT → the	3 [DT•,1]	3
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Reduce: [4]	NN → man	5 [DT NN •, 2]	5
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Axiom		1 [•,0]	1
Shift: [1]		2 [the•,1]	2
Reduce: [2]	DT → the	3 [DT•,1]	3
Shift: [3]		4 [DT man •, 2]	4
Reduce: [4]	NN → man	5 [DT NN •, 2]	5
Reduce: [5]	NP → DT NN	6 [NP •, 2]	6
Shift: [6]		7 [NP sleeps •, 3]	7
Reduce: [7]	Vi → sleeps	8 [NP Vi •, 3]	8
Reduce: [8]	VP → Vi	9 [NP VP •, 3]	9

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Reduce: [2]	DT → the	3 [DT•,1]	3
Shift: [3]		4 [DT man •, 2]	4
Reduce: [4]	NN → man	5 [DT NN •, 2]	5
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Shift: [6]		7 [NP sleeps •, 3]	7
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Shift: [1]		2 [the•,1]	2
Reduce: [2]	DT → the	3 [DT•,1]	3
Shift: [3]		4 [DT man •, 2]	4
Reduce: [4]	NN → man	5 [DT NN •, 2]	5
Reduce: [5]	NP → DT NN	6 [NP •, 2]	6
Shift: [6]		7 [NP sleeps •, 3]	7
Reduce: [7]	Vi → sleeps	8 [NP Vi •, 3]	8
Reduce: [8]	VP → Vi	9 [NP VP •, 3]	9
Reduce: [9]	S → NP VP	10 [S •, 3]	10

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Reduce: [2]	DT → the	3 [DT•,1]	3
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Reduce: [5]	NP → DT NN	6 [NP •, 2]	6
Shift: [6]		7 [NP sleeps •, 3]	7
Reduce: [7]	Vi → sleeps	8 [NP Vi •, 3]	8
Reduce: [8]	VP → Vi	9 [NP VP •, 3]	9
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GOAL: [10]			∅

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Shift-Reduce

Input: G and $w_1 \dots w_n$

Item form: $[\alpha \bullet, j]$
 asserts that $\alpha \Rightarrow^* w_1 \dots w_j$ or
 that $\alpha w_{j+1} \dots w_n \Rightarrow^* w_1 \dots w_j$

Axiom: $[\bullet, 0]$

Goal: $[S \bullet, n]$

Scan (shift)

asserts that $\alpha w_{j+1} \Rightarrow^* w_1 \dots w_j w_{j+1}$

Complete (reduce)

asserts that $\alpha B \Rightarrow^* w_1 \dots w_j$

$$\text{SHIFT} \frac{[\alpha \bullet, j]}{[\alpha w_{j+1}, j + 1]}$$

$$\text{REDUCE} \frac{[\alpha \gamma \bullet, j]}{[\alpha B \bullet, j]} \quad B \rightarrow \gamma \in R$$

Top-Down recognition

Input: G and $w_1 \dots w_n$

Item form: $[\bullet\beta, j]$
asserts that $S \Rightarrow^* w_1 \dots w_j \beta$

Axiom: $[\bullet S, 0]$

Goal: $[\bullet, n]$

Scan

asserts that $S \Rightarrow^* w_1 \dots w_j w_{j+1} \beta$

$$\text{SCAN} \frac{[\bullet w_{j+1} \beta, j]}{[\bullet \beta, j + 1]}$$

Predict

asserts that $S \Rightarrow^* w_1 \dots w_j B \beta$

$$\text{PREDICT} \frac{\bullet B \beta, j}{[\bullet \gamma \beta, j]} B \rightarrow \gamma \in R$$

Top-Down Example

Input: *the man sleeps*

S → NP VP

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Top-Down Example

Input: *the man sleeps*

Rule	Condition	Statement	Queue
Axiom	1	[• S, 0]	1

S → NP VP

VP → Vi

VP → Vt NP

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NP → DT NN

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Predict: [1]	S → NP VP	2	[• NP VP, 0]	2

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Top-Down Example

Input: *the man sleeps*

Rule	Condition	Statement	Queue	
Axiom		1	[• S, 0]	1
Predict: [1]	S → NP VP	2	[• NP VP, 0]	2
Predict: [2]	NP → DT NN	3	[• DT NN VP, 0]	3

S → NP VP 

VP → Vi

VP → Vt NP

~~VP → VP PP~~

NP → DT NN 

NP → NP PP

PP → IN NP

Vi → sleeps

Vt → saw

NN → man

NN → dog

NN → telescope

DT → the

IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition	Statement	Queue	
Axiom		1	[• S, 0]	1
Predict: [1]	$S \rightarrow NP VP$	2	[• NP VP, 0]	2
Predict: [2]	$NP \rightarrow DT NN$	3	[• DT NN VP, 0]	3

$S \rightarrow NP VP$ ←

$VP \rightarrow Vi$

$VP \rightarrow Vt NP$

~~$VP \rightarrow VP PP$~~

$NP \rightarrow DT NN$ ←

$NP \rightarrow NP PP$

$PP \rightarrow IN NP$

$Vi \rightarrow sleeps$

$Vt \rightarrow saw$

$NN \rightarrow man$

$NN \rightarrow dog$

$NN \rightarrow telescope$

$DT \rightarrow the$ ←

$IN \rightarrow with$

Top-Down Example

Input: *the man sleeps*

Rule	Condition	Statement	Queue
Axiom		1 [• S, 0]	1
Predict: [1]	S → NP VP	2 [• NP VP, 0]	2
Predict: [2]	NP → DT NN	3 [• DT NN VP, 0]	3
Predict: [3]	DT → the	4 [• the NN VP, 0]	4

S → NP VP

VP → Vi

VP → Vt NP

~~VP → VP PP~~

NP → DT NN

NP → NP PP

PP → IN NP

Vi → sleeps

Vt → saw

NN → man

NN → dog

NN → telescope

DT → the

IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition	Statement	Queue
Axiom		1 [• S, 0]	1
Predict: [1]	S → NP VP	2 [• NP VP, 0]	2
Predict: [2]	NP → DT NN	3 [• DT NN VP, 0]	3
Predict: [3]	DT → the	4 [• the NN VP, 0]	4

S → NP VP

VP → Vi

VP → Vt NP

~~VP → VP PP~~

NP → DT NN

NP → NP PP

PP → IN NP

Vi → sleeps

Vt → saw

NN → man

NN → dog

NN → telescope

DT → the

IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition	Statement	Queue
Axiom		1 [• S, 0]	1
Predict: [1]	S → NP VP	2 [• NP VP, 0]	2
Predict: [2]	NP → DT NN	3 [• DT NN VP, 0]	3
Predict: [3]	DT → the	4 [• the NN VP, 0]	4
Scan: [4]		5 [• NN VP, 1]	5

S → NP VP

VP → Vi

VP → Vt NP

~~VP → VP PP~~

NP → DT NN

NP → NP PP

PP → IN NP

Vi → sleeps

Vt → saw

NN → man

NN → dog

NN → telescope

DT → the

IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition	Statement	Queue
Axiom		1 [• S, 0]	1
Predict: [1]	S → NP VP	2 [• NP VP, 0]	2
Predict: [2]	NP → DT NN	3 [• DT NN VP, 0]	3
Predict: [3]	DT → the	4 [• the NN VP, 0]	4
Scan: [4]		5 [• NN VP, 1]	5

S → NP VP

VP → Vi

VP → Vt NP

~~VP → VP PP~~

NP → DT NN

NP → NP PP

PP → IN NP

Vi → sleeps

Vt → saw

NN → man

NN → dog

NN → telescope

DT → the

IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition	Statement	Queue
Axiom		1 [• S, 0]	1
Predict: [1]	S → NP VP	2 [• NP VP, 0]	2
Predict: [2]	NP → DT NN	3 [• DT NN VP, 0]	3
Predict: [3]	DT → the	4 [• the NN VP, 0]	4
Scan: [4]		5 [• NN VP, 1]	5
Predict: [5]	NN → man	6 [• man VP, 1]	6

S → NP VP

VP → Vi

VP → Vt NP

~~VP → VP PP~~

NP → DT NN

NP → NP PP

PP → IN NP

Vi → sleeps

Vt → saw

NN → man

NN → dog

NN → telescope

DT → the

IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition	Statement	Queue
Axiom		1 [• S, 0]	1
Predict: [1]	S → NP VP	2 [• NP VP, 0]	2
Predict: [2]	NP → DT NN	3 [• DT NN VP, 0]	3
Predict: [3]	DT → the	4 [• the NN VP, 0]	4
Scan: [4]		5 [• NN VP, 1]	5
Predict: [5]	NN → man	6 [• man VP, 1]	6

S → NP VP

VP → Vi

VP → Vt NP

~~VP → VP PP~~

NP → DT NN

NP → NP PP

PP → IN NP

Vi → sleeps

Vt → saw

NN → man

NN → dog

NN → telescope

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IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition	Statement	Queue
Axiom		1 [• S, 0]	1
Predict: [1]	S → NP VP	2 [• NP VP, 0]	2
Predict: [2]	NP → DT NN	3 [• DT NN VP, 0]	3
Predict: [3]	DT → the	4 [• the NN VP, 0]	4
Scan: [4]		5 [• NN VP, 1]	5
Predict: [5]	NN → man	6 [• man VP, 1]	6
Scan: [6]		7 [• VP, 2]	7

S → NP VP

VP → Vi

VP → Vt NP

~~VP → VP PP~~

NP → DT NN

NP → NP PP

PP → IN NP

Vi → sleeps

Vt → saw

NN → man

NN → dog

NN → telescope

DT → the

IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition	Statement	Queue
Axiom		[• S, 0]	1
Predict: [1]	S → NP VP	[• NP VP, 0]	2
Predict: [2]	NP → DT NN	[• DT NN VP, 0]	3
Predict: [3]	DT → the	[• the NN VP, 0]	4
Scan: [4]		[• NN VP, 1]	5
Predict: [5]	NN → man	[• man VP, 1]	6
Scan: [6]		[• VP, 2]	7

S → NP VP

VP → Vi

VP → Vt NP

~~VP → VP PP~~

NP → DT NN

NP → NP PP

PP → IN NP

Vi → sleeps

Vt → saw

NN → man

NN → dog

NN → telescope

DT → the

IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition	Statement	Queue
Axiom		[• S, 0]	1
Predict: [1]	S → NP VP	[• NP VP, 0]	2
Predict: [2]	NP → DT NN	[• DT NN VP, 0]	3
Predict: [3]	DT → the	[• the NN VP, 0]	4
Scan: [4]		[• NN VP, 1]	5
Predict: [5]	NN → man	[• man VP, 1]	6
Scan: [6]		[• VP, 2]	7

S → NP VP

VP → Vi

VP → Vt NP

~~VP → VP PP~~

NP → DT NN

NP → NP PP

PP → IN NP

Vi → sleeps

Vt → saw

NN → man

NN → dog

NN → telescope

DT → the

IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition	Statement	Queue
Axiom		1 [• S, 0]	1
Predict: [1]	S → NP VP	2 [• NP VP, 0]	2
Predict: [2]	NP → DT NN	3 [• DT NN VP, 0]	3
Predict: [3]	DT → the	4 [• the NN VP, 0]	4
Scan: [4]		5 [• NN VP, 1]	5
Predict: [5]	NN → man	6 [• man VP, 1]	6
Scan: [6]		7 [• VP, 2]	7
Predict: [7]	VP → Vi	8 [• Vi, 2]	8, 9

S → NP VP

VP → Vi

VP → Vt NP

~~VP → VP PP~~

NP → DT NN

NP → NP PP

PP → IN NP

Vi → sleeps

Vt → saw

NN → man

NN → dog

NN → telescope

DT → the

IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition	Statement	Queue
Axiom		[• S, 0]	1
Predict: [1]	S → NP VP	[• NP VP, 0]	2
Predict: [2]	NP → DT NN	[• DT NN VP, 0]	3
Predict: [3]	DT → the	[• the NN VP, 0]	4
Scan: [4]		[• NN VP, 1]	5
Predict: [5]	NN → man	[• man VP, 1]	6
Scan: [6]		[• VP, 2]	7
Predict: [7]	VP → Vi	[• Vi, 2]	8, 9
	VP → Vt NP	[• Vt NP, 2]	

S → NP VP

VP → Vi

VP → Vt NP

~~VP → VP PP~~

NP → DT NN

NP → NP PP

PP → IN NP

Vi → sleeps

Vt → saw

NN → man

NN → dog

NN → telescope

DT → the

IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition	Statement	Queue
Axiom		[• S, 0]	1
Predict: [1]	S → NP VP	[• NP VP, 0]	2
Predict: [2]	NP → DT NN	[• DT NN VP, 0]	3
Predict: [3]	DT → the	[• the NN VP, 0]	4
Scan: [4]		[• NN VP, 1]	5
Predict: [5]	NN → man	[• man VP, 1]	6
Scan: [6]		[• VP, 2]	7
Predict: [7]	VP → Vi	[• Vi, 2]	8, 9
	VP → Vt NP	[• Vt NP, 2]	

S → NP VP

VP → Vi

VP → Vt NP

~~VP → VP PP~~

NP → DT NN

NP → NP PP

PP → IN NP

Vi → sleeps

Vt → saw

NN → man

NN → dog

NN → telescope

DT → the

IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition	Statement	Queue
Axiom		[• S, 0]	1
Predict: [1]	S → NP VP	[• NP VP, 0]	2
Predict: [2]	NP → DT NN	[• DT NN VP, 0]	3
Predict: [3]	DT → the	[• the NN VP, 0]	4
Scan: [4]		[• NN VP, 1]	5
Predict: [5]	NN → man	[• man VP, 1]	6
Scan: [6]		[• VP, 2]	7
Predict: [7]	VP → Vi	[• Vi, 2]	8, 9
	VP → Vt NP	[• Vt NP, 2]	
Predict: [8]	Vi → sleeps	[• sleeps, 2]	9, 10

S → NP VP

VP → Vi

VP → Vt NP

~~VP → VP PP~~

NP → DT NN

NP → NP PP

PP → IN NP

Vi → sleeps

Vt → saw

NN → man

NN → dog

NN → telescope

DT → the

IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition	Statement	Queue
Axiom		[• S, 0]	1
Predict: [1]	S → NP VP	[• NP VP, 0]	2
Predict: [2]	NP → DT NN	[• DT NN VP, 0]	3
Predict: [3]	DT → the	[• the NN VP, 0]	4
Scan: [4]		[• NN VP, 1]	5
Predict: [5]	NN → man	[• man VP, 1]	6
Scan: [6]		[• VP, 2]	7
Predict: [7]	VP → Vi	[• Vi, 2]	8, 9
	VP → Vt NP	[• Vt NP, 2]	
Predict: [8]	Vi → sleeps	[• sleeps, 2]	9, 10

S → NP VP

VP → Vi

VP → Vt NP

~~VP → VP PP~~

NP → DT NN

NP → NP PP

PP → IN NP

Vi → sleeps

Vt → saw

NN → man

NN → dog

NN → telescope

DT → the

IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition	Statement	Queue
Axiom		1 [• S, 0]	1
Predict: [1]	S → NP VP	2 [• NP VP, 0]	2
Predict: [2]	NP → DT NN	3 [• DT NN VP, 0]	3
Predict: [3]	DT → the	4 [• the NN VP, 0]	4
Scan: [4]		5 [• NN VP, 1]	5
Predict: [5]	NN → man	6 [• man VP, 1]	6
Scan: [6]		7 [• VP, 2]	7
Predict: [7]	VP → Vi	8 [• Vi, 2]	8, 9
	VP → Vt NP	9 [• Vt NP, 2]	
Predict: [8]	Vi → sleeps	10 [• sleeps, 2]	9, 10
			10

S → NP VP

VP → Vi

VP → Vt NP

~~VP → VP PP~~

NP → DT NN

NP → NP PP

PP → IN NP

Vi → sleeps

Vt → saw

NN → man

NN → dog

NN → telescope

DT → the

IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition	Statement	Queue
Axiom		1 [• S, 0]	1
Predict: [1]	S → NP VP	2 [• NP VP, 0]	2
Predict: [2]	NP → DT NN	3 [• DT NN VP, 0]	3
Predict: [3]	DT → the	4 [• the NN VP, 0]	4
Scan: [4]		5 [• NN VP, 1]	5
Predict: [5]	NN → man	6 [• man VP, 1]	6
Scan: [6]		7 [• VP, 2]	7
Predict: [7]	VP → Vi	8 [• Vi, 2]	8, 9
	VP → Vt NP	9 [• Vt NP, 2]	
Predict: [8]	Vi → sleeps	10 [• sleeps, 2]	9, 10
			10

S → NP VP

VP → Vi

VP → Vt NP

~~VP → VP PP~~

NP → DT NN

NP → NP PP

PP → IN NP

Vi → sleeps

Vt → saw

NN → man

NN → dog

NN → telescope

DT → the

IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition	Statement	Queue
Axiom		[• S, 0]	1
Predict: [1]	S → NP VP	[• NP VP, 0]	2
Predict: [2]	NP → DT NN	[• DT NN VP, 0]	3
Predict: [3]	DT → the	[• the NN VP, 0]	4
Scan: [4]		[• NN VP, 1]	5
Predict: [5]	NN → man	[• man VP, 1]	6
Scan: [6]		[• VP, 2]	7
Predict: [7]	VP → Vi	[• Vi, 2]	8, 9
	VP → Vt NP	[• Vt NP, 2]	
Predict: [8]	Vi → sleeps	[• sleeps, 2]	9, 10
			10
Scan: [10]		[•, 3]	11

S → NP VP

VP → Vi

VP → Vt NP

~~VP → VP PP~~

NP → DT NN

NP → NP PP

PP → IN NP

Vi → sleeps

Vt → saw

NN → man

NN → dog

NN → telescope

DT → the

IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition	Statement	Queue
Axiom		1 [• S, 0]	1
Predict: [1]	S → NP VP	2 [• NP VP, 0]	2
Predict: [2]	NP → DT NN	3 [• DT NN VP, 0]	3
Predict: [3]	DT → the	4 [• the NN VP, 0]	4
Scan: [4]		5 [• NN VP, 1]	5
Predict: [5]	NN → man	6 [• man VP, 1]	6
Scan: [6]		7 [• VP, 2]	7
Predict: [7]	VP → Vi	8 [• Vi, 2]	8, 9
	VP → Vt NP	9 [• Vt NP, 2]	
Predict: [8]	Vi → sleeps	10 [• sleeps, 2]	9, 10
			10
Scan: [10]		11 [•, 3]	11
GOAL: [11]			∅

S → NP VP

VP → Vi

VP → Vt NP

~~VP → VP PP~~

NP → DT NN

NP → NP PP

PP → IN NP

Vi → sleeps

Vt → saw

NN → man

NN → dog

NN → telescope

DT → the

IN → with

CKY - CNF only

Input: G and $s = w_1 \dots w_n$ **Item form:** $[i, X, j]$
asserts that $X \Rightarrow^* w_{i+1} \dots w_j$

Axioms: $[i, X, i+1] \quad X \rightarrow w_i \in R$

Goal: $[0, S, n]$

Merge:
$$\frac{[i, A, k] \quad [k, B, j]}{[i, C, j]} \quad C \rightarrow AB \in R$$

asserts that

$w_{i+1} \dots w_k w_{k+1} \dots w_j \Rightarrow^* w_{i+1} \dots w_j$

CKY Example

Input: *the man saw the dog*

S → NP VP

Vi → sleeps

~~VP → Vi~~

Vt → saw

VP → Vt NP

NN → man

VP → VP PP

NN → dog

NP → DT NN

NN → telescope

NP → NP PP

DT → the

PP → IN NP

IN → with

CKY Example

Input: *the man saw the dog*

S → NP VP

Vi → sleeps

~~VP → Vi~~

Vt → saw

VP → Vt NP

NN → man

VP → VP PP

NN → dog

NP → DT NN

NN → telescope

NP → NP PP

DT → the

PP → IN NP

IN → with

Rule

Condition

Statement

Queue

Passive

CKY Example

Input: *the man saw the dog*

$S \rightarrow NP VP$	$V_i \rightarrow \text{sleeps}$
$VP \rightarrow V_i$	$V_t \rightarrow \text{saw}$
$VP \rightarrow V_t NP$	$NN \rightarrow \text{man}$
$VP \rightarrow VP PP$	$NN \rightarrow \text{dog}$
$NP \rightarrow DT NN$	$NN \rightarrow \text{telescope}$
$NP \rightarrow NP PP$	$DT \rightarrow \text{the}$
$PP \rightarrow IN NP$	$IN \rightarrow \text{with}$

Rule	Condition	Statement	Queue	Passive
Axiom	$DT \rightarrow \text{the}$	1 [0, DT, 1]	1	

CKY Example

Input: *the man saw the dog*

S → NP VP	Vi → sleeps
VP → Vi	Vt → saw
VP → Vt NP	NN → man
VP → VP PP	NN → dog
NP → DT NN	NN → telescope
NP → NP PP	DT → the
PP → IN NP	IN → with

Rule	Condition	Statement	Queue	Passive
Axiom	DT → the	1 [0, DT, 1]	1	
	NN → man	2 [1, NN, 2]	1, 2	

CKY Example

Input: *the man saw the dog*

S → NP VP	Vi → sleeps
VP → Vi	Vt → saw
VP → Vt NP	NN → man
VP → VP PP	NN → dog
NP → DT NN	NN → telescope
NP → NP PP	DT → the
PP → IN NP	IN → with

Rule	Condition	Statement	Queue	Passive
Axiom	DT → the	1 [0, DT, 1]	1	
	NN → man	2 [1, NN, 2]	1, 2	
	Vt → saw	3 [2, Vt, 3]	1, 2, 3	

CKY Example

Input: *the man saw the dog*

S → NP VP	Vi → sleeps
VP → Vi	Vt → saw
VP → Vt NP	NN → man
VP → VP PP	NN → dog
NP → DT NN	NN → telescope
NP → NP PP	DT → the
PP → IN NP	IN → with

Rule	Condition	Statement	Queue	Passive
Axiom	DT → the	1 [0, DT, 1]	1	
	NN → man	2 [1, NN, 2]	1, 2	
	Vt → saw	3 [2, Vt, 3]	1, 2, 3	
	DT → the	4 [3, DT, 4]	1, 2, 3, 4	

CKY Example

Input: *the man saw the dog*

S → NP VP	Vi → sleeps
VP → Vi	Vt → saw
VP → Vt NP	NN → man
VP → VP PP	NN → dog
NP → DT NN	NN → telescope
NP → NP PP	DT → the
PP → IN NP	IN → with

Rule	Condition	Statement	Queue	Passive
Axiom	DT → the	1 [0, DT, 1]	1	
	NN → man	2 [1, NN, 2]	1, 2	
	Vt → saw	3 [2, Vt, 3]	1, 2, 3	
	DT → the	4 [3, DT, 4]	1, 2, 3, 4	
	NN → dog	5 [4, NN, 5]	1, 2, 3, 4, 5	

CKY Example

Input: *the man saw the dog*

S → NP VP	Vi → sleeps
VP → Vi	Vt → saw
VP → Vt NP	NN → man
VP → VP PP	NN → dog
NP → DT NN	NN → telescope
NP → NP PP	DT → the
PP → IN NP	IN → with

Rule	Condition	Statement	Queue	Passive
Axiom	DT → the	1 [0, DT, 1]	1	
	NN → man	2 [1, NN, 2]	1, 2	
	Vt → saw	3 [2, Vt, 3]	1, 2, 3	
	DT → the	4 [3, DT, 4]	1, 2, 3, 4	
	NN → dog	5 [4, NN, 5]	1, 2, 3, 4, 5	
			2, 3, 4, 5	1

CKY Example

Input: *the man saw the dog*

S → NP VP	Vi → sleeps
VP → Vi	Vt → saw
VP → Vt NP	NN → man
VP → VP PP	NN → dog
NP → DT NN	NN → telescope
NP → NP PP	DT → the
PP → IN NP	IN → with

Rule	Condition	Statement	Queue	Passive
Axiom	DT → the	1 [0, DT, 1]	1	
	NN → man	2 [1, NN, 2]	1, 2	
	Vt → saw	3 [2, Vt, 3]	1, 2, 3	
	DT → the	4 [3, DT, 4]	1, 2, 3, 4	
	NN → dog	5 [4, NN, 5]	1, 2, 3, 4, 5	
			2, 3, 4, 5	1
Merge: [1][2]	NP → DT NN	6 [0, NP, 2]	3, 4, 5, 6	2

CKY Example

Input: *the man saw the dog*

S → NP VP	Vi → sleeps
VP → Vi	Vt → saw
VP → Vt NP	NN → man
VP → VP PP	NN → dog
NP → DT NN	NN → telescope
NP → NP PP	DT → the
PP → IN NP	IN → with

Rule	Condition	Statement	Queue	Passive
Axiom	DT → the	1 [0, DT, 1]	1	
	NN → man	2 [1, NN, 2]	1, 2	
	Vt → saw	3 [2, Vt, 3]	1, 2, 3	
	DT → the	4 [3, DT, 4]	1, 2, 3, 4	
	NN → dog	5 [4, NN, 5]	1, 2, 3, 4, 5	
			2, 3, 4, 5	1
Merge: [1][2]	NP → DT NN	6 [0, NP, 2]	3, 4, 5, 6	2
			4, 5, 6	3

CKY Example

Input: *the man saw the dog*

S → NP VP	Vi → sleeps
VP → Vi	Vt → saw
VP → Vt NP	NN → man
VP → VP PP	NN → dog
NP → DT NN	NN → telescope
NP → NP PP	DT → the
PP → IN NP	IN → with

Rule	Condition	Statement	Queue	Passive
Axiom	DT → the	1 [0, DT, 1]	1	
	NN → man	2 [1, NN, 2]	1, 2	
	Vt → saw	3 [2, Vt, 3]	1, 2, 3	
	DT → the	4 [3, DT, 4]	1, 2, 3, 4	
	NN → dog	5 [4, NN, 5]	1, 2, 3, 4, 5	
			2, 3, 4, 5	1
Merge: [1][2]	NP → DT NN	6 [0, NP, 2]	3, 4, 5, 6	2
			4, 5, 6	3
			5, 6	4

CKY Example

Input: *the man saw the dog*

S → NP VP	Vi → sleeps
VP → Vi	Vt → saw
VP → Vt NP	NN → man
VP → VP PP	NN → dog
NP → DT NN	NN → telescope
NP → NP PP	DT → the
PP → IN NP	IN → with

Rule	Condition	Statement	Queue	Passive
Axiom	DT → the	1 [0, DT, 1]	1	
	NN → man	2 [1, NN, 2]	1, 2	
	Vt → saw	3 [2, Vt, 3]	1, 2, 3	
	DT → the	4 [3, DT, 4]	1, 2, 3, 4	
	NN → dog	5 [4, NN, 5]	1, 2, 3, 4, 5	
			2, 3, 4, 5	1
Merge: [1][2]	NP → DT NN	6 [0, NP, 2]	3, 4, 5, 6	2
			4, 5, 6	3
			5, 6	4
Merge: [4][5]	NP → DT NN	7 [3, NP, 5]	6, 7	5

CKY Example

Input: *the man saw the dog*

S → NP VP	Vi → sleeps
VP → Vi	Vt → saw
VP → Vt NP	NN → man
VP → VP PP	NN → dog
NP → DT NN	NN → telescope
NP → NP PP	DT → the
PP → IN NP	IN → with

Rule	Condition	Statement	Queue	Passive
Axiom	DT → the	1 [0, DT, 1]	1	
	NN → man	2 [1, NN, 2]	1, 2	
	Vt → saw	3 [2, Vt, 3]	1, 2, 3	
	DT → the	4 [3, DT, 4]	1, 2, 3, 4	
	NN → dog	5 [4, NN, 5]	1, 2, 3, 4, 5	
			2, 3, 4, 5	1
Merge: [1][2]	NP → DT NN	6 [0, NP, 2]	3, 4, 5, 6	2
			4, 5, 6	3
			5, 6	4
Merge: [4][5]	NP → DT NN	7 [3, NP, 5]	6, 7	5
			7	6

CKY Example

Input: *the man saw the dog*

S → NP VP	Vi → sleeps
VP → Vi	Vt → saw
VP → Vt NP	NN → man
VP → VP PP	NN → dog
NP → DT NN	NN → telescope
NP → NP PP	DT → the
PP → IN NP	IN → with

Rule	Condition	Statement	Queue	Passive
Axiom	DT → the	1 [0, DT, 1]	1	
	NN → man	2 [1, NN, 2]	1, 2	
	Vt → saw	3 [2, Vt, 3]	1, 2, 3	
	DT → the	4 [3, DT, 4]	1, 2, 3, 4	
	NN → dog	5 [4, NN, 5]	1, 2, 3, 4, 5	
			2, 3, 4, 5	1
Merge: [1][2]	NP → DT NN	6 [0, NP, 2]	3, 4, 5, 6	2
			4, 5, 6	3
			5, 6	4
Merge: [4][5]	NP → DT NN	7 [3, NP, 5]	6, 7	5
			7	6
Merge: [3][7]	VP → Vt NP	8 [2, VP, 5]	8	7

CKY Example

Input: *the man saw the dog*

S → NP VP	Vi → sleeps
VP → Vi	Vt → saw
VP → Vt NP	NN → man
VP → VP PP	NN → dog
NP → DT NN	NN → telescope
NP → NP PP	DT → the
PP → IN NP	IN → with

Rule	Condition	Statement	Queue	Passive
Axiom	DT → the	1 [0, DT, 1]	1	
	NN → man	2 [1, NN, 2]	1, 2	
	Vt → saw	3 [2, Vt, 3]	1, 2, 3	
	DT → the	4 [3, DT, 4]	1, 2, 3, 4	
	NN → dog	5 [4, NN, 5]	1, 2, 3, 4, 5	
			2, 3, 4, 5	1
Merge: [1][2]	NP → DT NN	6 [0, NP, 2]	3, 4, 5, 6	2
			4, 5, 6	3
			5, 6	4
Merge: [4][5]	NP → DT NN	7 [3, NP, 5]	6, 7	5
			7	6
Merge: [3][7]	VP → Vt NP	8 [2, VP, 5]	8	7
Merge: [6][8]	S → NP VP	9 [0, S, 5]	9	8

CKY Example

Input: *the man saw the dog*

S → NP VP	Vi → sleeps
VP → Vi	Vt → saw
VP → Vt NP	NN → man
VP → VP PP	NN → dog
NP → DT NN	NN → telescope
NP → NP PP	DT → the
PP → IN NP	IN → with

Rule	Condition	Statement	Queue	Passive
Axiom	DT → the	1 [0, DT, 1]	1	
	NN → man	2 [1, NN, 2]	1, 2	
	Vt → saw	3 [2, Vt, 3]	1, 2, 3	
	DT → the	4 [3, DT, 4]	1, 2, 3, 4	
	NN → dog	5 [4, NN, 5]	1, 2, 3, 4, 5	
			2, 3, 4, 5	1
Merge: [1][2]	NP → DT NN	6 [0, NP, 2]	3, 4, 5, 6	2
			4, 5, 6	3
			5, 6	4
Merge: [4][5]	NP → DT NN	7 [3, NP, 5]	6, 7	5
			7	6
Merge: [3][7]	VP → Vt NP	8 [2, VP, 5]	8	7
Merge: [6][8]	S → NP VP	9 [0, S, 5]	9	8
GOAL: [9]			∅	9

Rule Segmentation: "Split Points"

${}_0S_3 \rightarrow {}_0NP_2 {}_2VP_3$

${}_0NP_2 \rightarrow {}_0DT_1 {}_1NN_2$

${}_2VP_3 \rightarrow {}_2Vi_3$

${}_0DT_1 \rightarrow \text{the}$

${}_1NN_2 \rightarrow \text{man}$

${}_2Vi_3 \rightarrow \text{sleeps}$

0 1 2 3

Rule Segmentation: "Split Points"

0 the 1 man 2 sleeps 3

${}_0S_3 \rightarrow {}_0NP_2 {}_2VP_3$

${}_0NP_2 \rightarrow {}_0DT_1 {}_1NN_2$

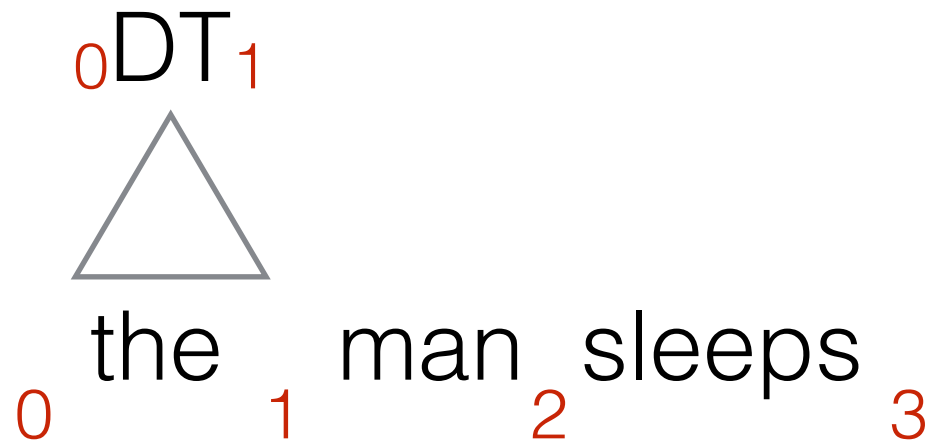
${}_2VP_3 \rightarrow {}_2Vi_3$

${}_0DT_1 \rightarrow \text{the}$

${}_1NN_2 \rightarrow \text{man}$

${}_2Vi_3 \rightarrow \text{sleeps}$

Rule Segmentation: "Split Points"



$0S_3 \rightarrow 0NP_2 2VP_3$

$0NP_2 \rightarrow 0DT_1 1NN_2$

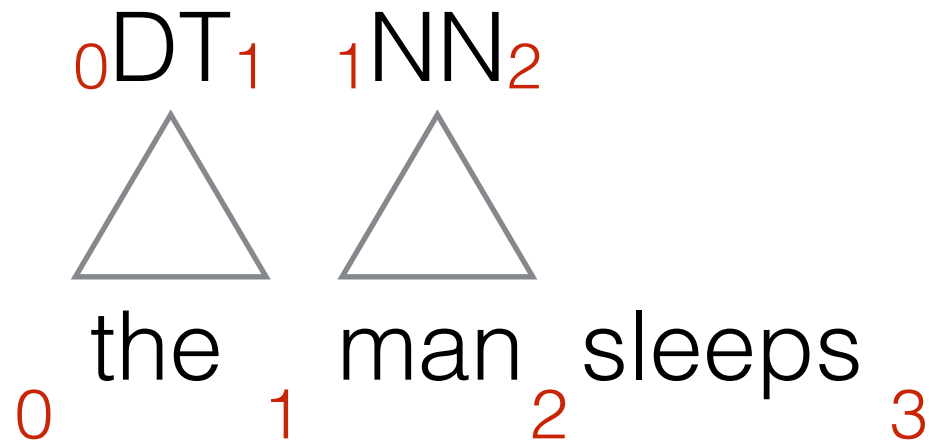
$2VP_3 \rightarrow 2Vi_3$

$0DT_1 \rightarrow$ the

$1NN_2 \rightarrow$ man

$2Vi_3 \rightarrow$ sleeps

Rule Segmentation: "Split Points"



0S₃ → 0NP₂ 2VP₃

0NP₂ → 0DT₁ 1NN₂

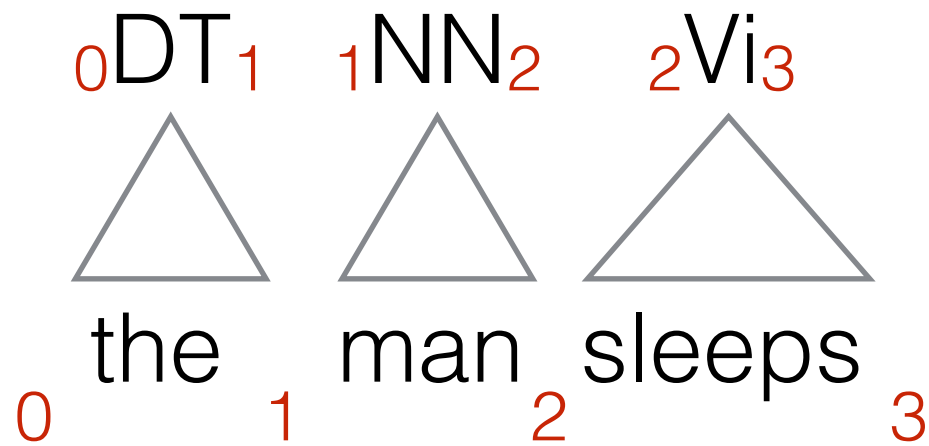
2VP₃ → 2Vi₃

0DT₁ → the

1NN₂ → man

2Vi₃ → sleeps

Rule Segmentation: "Split Points"



${}_0S_3 \rightarrow {}_0NP_2 {}_2VP_3$

${}_0NP_2 \rightarrow {}_0DT_1 {}_1NN_2$

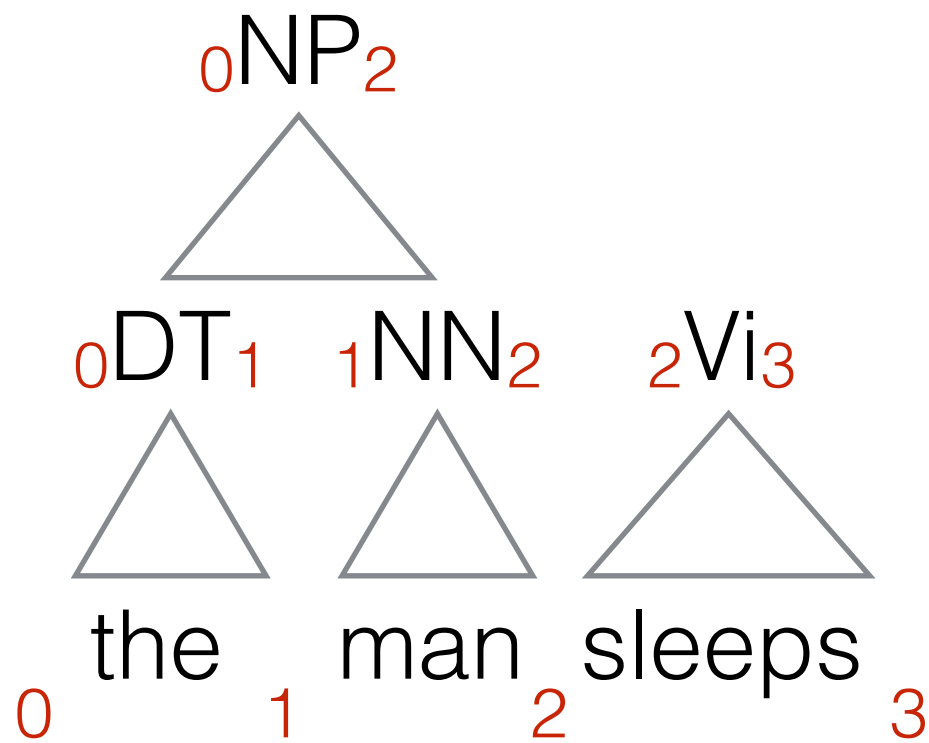
${}_2VP_3 \rightarrow {}_2Vi_3$

${}_0DT_1 \rightarrow \text{the}$

${}_1NN_2 \rightarrow \text{man}$

${}_2Vi_3 \rightarrow \text{sleeps}$

Rule Segmentation: "Split Points"



0 S₃ → 0 NP₂ 2 VP₃

0 NP₂ → 0 DT₁ 1 NN₂

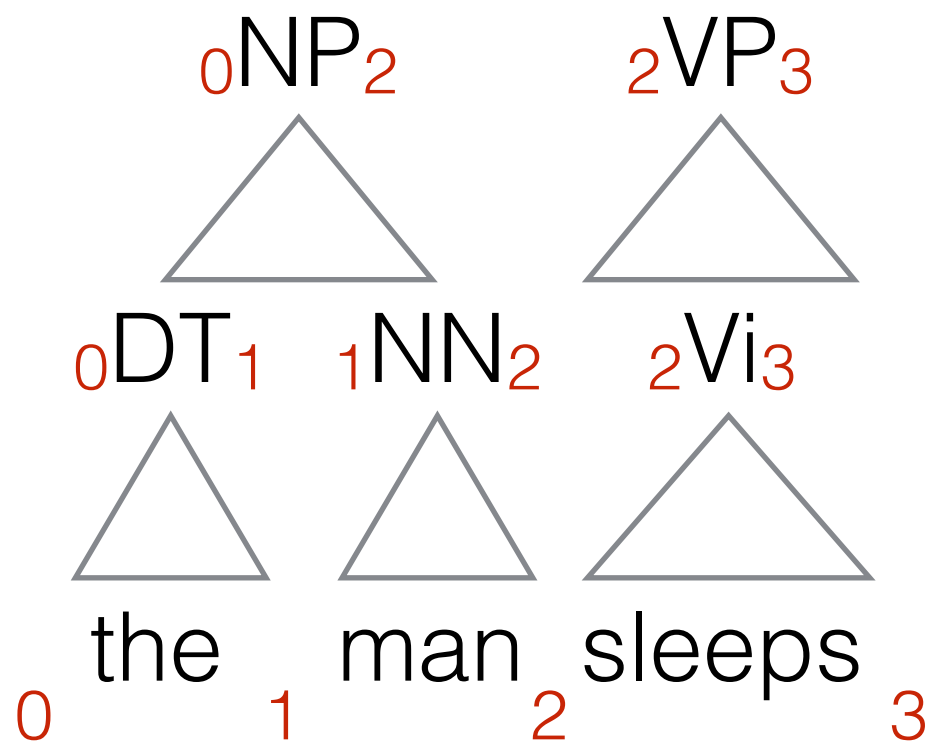
2 VP₃ → 2 Vi₃

0 DT₁ → the

1 NN₂ → man

2 Vi₃ → sleeps

Rule Segmentation: "Split Points"



0 S₃ → 0 NP₂ 2 VP₃

0 NP₂ → 0 DT₁ 1 NN₂

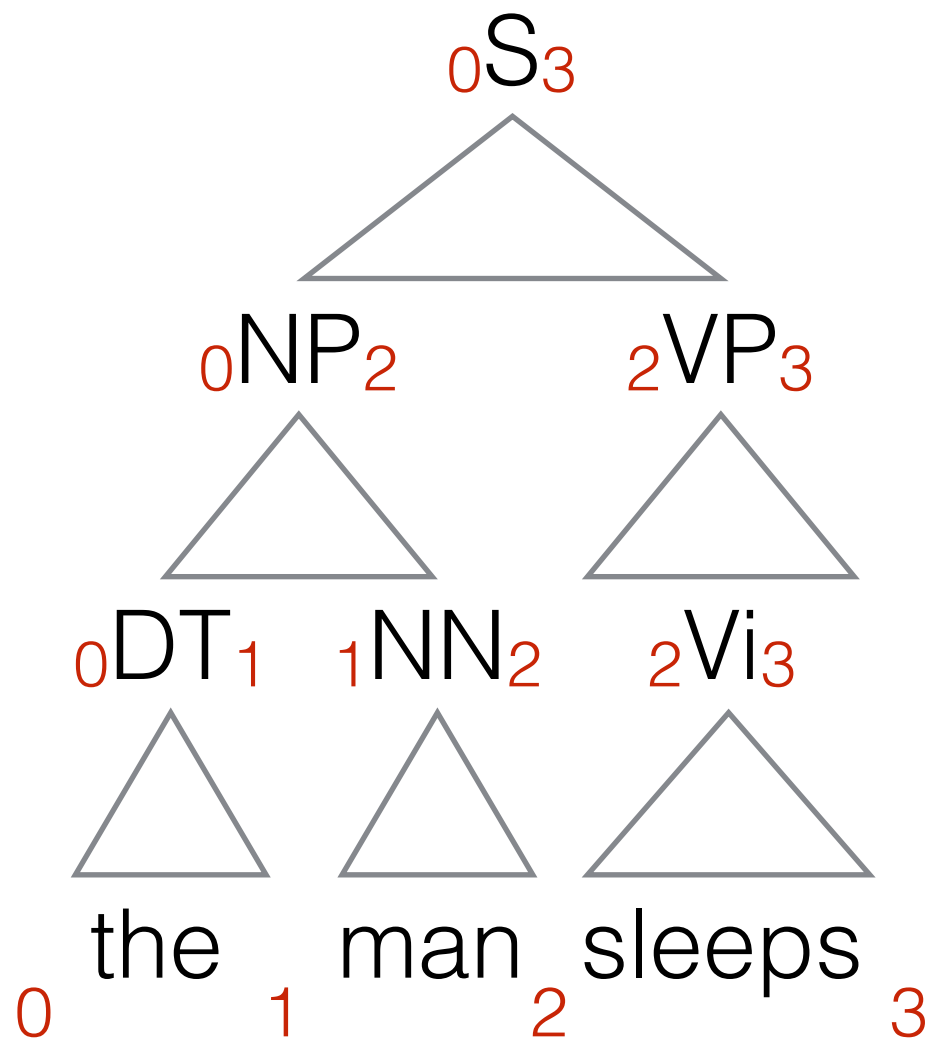
2 VP₃ → 2 Vi₃

0 DT₁ → the

1 NN₂ → man

2 Vi₃ → sleeps

Rule Segmentation: "Split Points"



0 S₃ → 0 NP₂ 2 VP₃

0 NP₂ → 0 DT₁ 1 NN₂

2 VP₃ → 2 Vi₃

0 DT₁ → the

1 NN₂ → man

2 Vi₃ → sleeps

"Dotted items"

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Parsing a CNF grammar is easy because we know the shape of rules

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When that's not the case, we have to **scan rules symbol by symbol** using a general mechanism:

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- The dot represents progress through the rule's right-hand side (RHS)

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- The filled box represents a segmentation of $[0 .. j]$ into $|\alpha|$ adjacent parts

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- In general, we segment rules with respect to the input $w_1 \dots w_n$
- The dot represents progress through the rule's right-hand side (RHS)
- The prefix α has already been parsed and we are waiting for β
- The filled box represents a segmentation of $[0 .. j]$ into $|\alpha|$ adjacent parts
- The empty box has no actual role, it's just a reminder that the segmentation beyond j is unknown

CKY+

Input: G and $s = w_1 \dots w_n$

Item form: $[i, X \rightarrow \alpha \blacksquare \bullet \beta \square, j]$
 asserts that $X \Rightarrow^* w_{i+1} \dots w_j \beta$

Axioms: $[i, X \rightarrow w_i \bullet \alpha \square, i+1]$ $X \rightarrow w_i \alpha \in R$
 $[i, X \rightarrow \varepsilon \bullet, i]$ $X \rightarrow \varepsilon \in R$

Goal: $[0, S \rightarrow \alpha \blacksquare \bullet, n]$

Scan

Prefix

$$\frac{[i, X \rightarrow \alpha \blacksquare \bullet w_{j+1} \beta \square, j]}{[i, X \rightarrow \alpha \blacksquare w_{j+1} \bullet \beta \square, j+1]} \quad \frac{[i, Y \rightarrow \alpha \blacksquare \bullet, j]}{[i, X \rightarrow Y_{i,j} \bullet \beta \square, j]} \quad X \rightarrow Y \beta \in R$$

Complete

$$\frac{[i, X \rightarrow \alpha \blacksquare \bullet Y \beta \square, k] [k, Y \rightarrow \gamma \blacksquare \bullet, j]}{[i, X \rightarrow \alpha \blacksquare Y_{k,j} \bullet \beta \square, j]}$$

CKY + Example

Input: *the man sleeps*

S → NP VP

Vi → sleeps

VP → Vi

Vt → saw

VP → Vt NP

NN → man

VP → VP PP

NN → dog

NP → DT NN

NN → telescope

NP → NP PP

DT → the

PP → IN NP

IN → with

CKY + Example

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Rule

Condition

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Active

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NN → telescope

NP → NP PP

DT → the

PP → IN NP

IN → with

Rule	Condition	Item	Active	Passive
Axiom	DT → the	1 [0, DT → the •, 1]	1	

CKY + Example

Input: *the man sleeps*

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VP → Vi	Vt → saw
VP → Vt NP	NN → man
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NP → NP PP	DT → the
PP → IN NP	IN → with

Rule	Condition	Item	Active	Passive
Axiom	DT → the	1 [0, DT → the •, 1]	1	
	NN → man	2 [1, NN → man •, 2]	1, 2	

CKY + Example

Input: *the man sleeps*

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VP → Vi	Vt → saw
VP → Vt NP	NN → man
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Rule	Condition	Item	Active	Passive
Axiom	DT → the	1 [0, DT → the •, 1]	1	
	NN → man	2 [1, NN → man •, 2]	1, 2	
	Vi → sleeps	3 [2, Vi → sleeps •, 3]	1, 2, 3	

CKY + Example

Input: *the man sleeps*

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VP → Vi	Vt → saw
VP → Vt NP	NN → man
VP → VP PP	NN → dog
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Rule	Condition	Item	Active	Passive
Axiom	DT → the	1 [0, DT → the •, 1]	1	
	NN → man	2 [1, NN → man •, 2]	1, 2	
	Vi → sleeps	3 [2, Vi → sleeps •, 3]	1, 2, 3	
Prefix: [1]	NP → DT NN	4 [0, NP → DT _{0,1} • NN, 2]	2, 3, 4	1

CKY + Example

$S \rightarrow NP VP$	$Vi \rightarrow \text{sleeps}$
$VP \rightarrow Vi$	$Vt \rightarrow \text{saw}$
$VP \rightarrow Vt NP$	$NN \rightarrow \text{man}$
$VP \rightarrow VP PP$	$NN \rightarrow \text{dog}$
$NP \rightarrow DT NN$	$NN \rightarrow \text{telescope}$
$NP \rightarrow NP PP$	$DT \rightarrow \text{the}$
$PP \rightarrow IN NP$	$IN \rightarrow \text{with}$

Input: *the man sleeps*

Rule	Condition	Item	Active	Passive
Axiom	$DT \rightarrow \text{the}$	1 [0, $DT \rightarrow \text{the} \bullet$, 1]	1	
	$NN \rightarrow \text{man}$	2 [1, $NN \rightarrow \text{man} \bullet$, 2]	1, 2	
	$Vi \rightarrow \text{sleeps}$	3 [2, $Vi \rightarrow \text{sleeps} \bullet$, 3]	1, 2, 3	
Prefix: [1]	$NP \rightarrow DT NN$	4 [0, $NP \rightarrow DT_{0,1} \bullet NN$, 2]	2, 3, 4 3, 4	1 2

CKY + Example

Input: *the man sleeps*

$S \rightarrow NP VP$	$Vi \rightarrow \text{sleeps}$
$VP \rightarrow Vi$	$Vt \rightarrow \text{saw}$
$VP \rightarrow Vt NP$	$NN \rightarrow \text{man}$
$VP \rightarrow VP PP$	$NN \rightarrow \text{dog}$
$NP \rightarrow DT NN$	$NN \rightarrow \text{telescope}$
$NP \rightarrow NP PP$	$DT \rightarrow \text{the}$
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Rule	Condition	Item	Active	Passive
Axiom	$DT \rightarrow \text{the}$	1 [0, $DT \rightarrow \text{the} \bullet$, 1]	1	
	$NN \rightarrow \text{man}$	2 [1, $NN \rightarrow \text{man} \bullet$, 2]	1, 2	
	$Vi \rightarrow \text{sleeps}$	3 [2, $Vi \rightarrow \text{sleeps} \bullet$, 3]	1, 2, 3	
Prefix: [1]	$NP \rightarrow DT NN$	4 [0, $NP \rightarrow DT_{0,1} \bullet NN$, 2]	2, 3, 4	1
			3, 4	2
Prefix: [3]	$VP \rightarrow Vi$	5 [2, $VP \rightarrow Vi_{2,3} \bullet$, 3]	4, 5	3

CKY + Example

Input: *the man sleeps*

$S \rightarrow NP VP$	$Vi \rightarrow \text{sleeps}$
$VP \rightarrow Vi$	$Vt \rightarrow \text{saw}$
$VP \rightarrow Vt NP$	$NN \rightarrow \text{man}$
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Rule	Condition	Item	Active	Passive
Axiom	$DT \rightarrow \text{the}$	1 [0, $DT \rightarrow \text{the} \bullet, 1$]	1	
	$NN \rightarrow \text{man}$	2 [1, $NN \rightarrow \text{man} \bullet, 2$]	1, 2	
	$Vi \rightarrow \text{sleeps}$	3 [2, $Vi \rightarrow \text{sleeps} \bullet, 3$]	1, 2, 3	
Prefix: [1]	$NP \rightarrow DT NN$	4 [0, $NP \rightarrow DT_{0,1} \bullet NN, 2$]	2, 3, 4	1
			3, 4	2
Prefix: [3]	$VP \rightarrow Vi$	5 [2, $VP \rightarrow Vi_{2,3} \bullet, 3$]	4, 5	3
Complete: [4] [2]		6 [0, $NP \rightarrow DT_{0,1} NN_{1,2} \bullet, 2$]	5, 6	4

CKY + Example

Input: *the man sleeps*

$S \rightarrow NP VP$	$Vi \rightarrow \text{sleeps}$
$VP \rightarrow Vi$	$Vt \rightarrow \text{saw}$
$VP \rightarrow Vt NP$	$NN \rightarrow \text{man}$
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$NP \rightarrow DT NN$	$NN \rightarrow \text{telescope}$
$NP \rightarrow NP PP$	$DT \rightarrow \text{the}$
$PP \rightarrow IN NP$	$IN \rightarrow \text{with}$

Rule	Condition	Item	Active	Passive
Axiom	$DT \rightarrow \text{the}$	1 [0, $DT \rightarrow \text{the} \bullet, 1$]	1	
	$NN \rightarrow \text{man}$	2 [1, $NN \rightarrow \text{man} \bullet, 2$]	1, 2	
	$Vi \rightarrow \text{sleeps}$	3 [2, $Vi \rightarrow \text{sleeps} \bullet, 3$]	1, 2, 3	
Prefix: [1]	$NP \rightarrow DT NN$	4 [0, $NP \rightarrow DT_{0,1} \bullet NN, 2$]	2, 3, 4	1
			3, 4	2
Prefix: [3]	$VP \rightarrow Vi$	5 [2, $VP \rightarrow Vi_{2,3} \bullet, 3$]	4, 5	3
Complete: [4] [2]		6 [0, $NP \rightarrow DT_{0,1} NN_{1,2} \bullet, 2$]	5, 6	4
			6	5

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Input: *the man sleeps*

$S \rightarrow NP VP$	$Vi \rightarrow \text{sleeps}$
$VP \rightarrow Vi$	$Vt \rightarrow \text{saw}$
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$NP \rightarrow NP PP$	$DT \rightarrow \text{the}$
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Rule	Condition	Item	Active	Passive
Axiom	$DT \rightarrow \text{the}$	1 [0, $DT \rightarrow \text{the} \bullet, 1$]	1	
	$NN \rightarrow \text{man}$	2 [1, $NN \rightarrow \text{man} \bullet, 2$]	1, 2	
	$Vi \rightarrow \text{sleeps}$	3 [2, $Vi \rightarrow \text{sleeps} \bullet, 3$]	1, 2, 3	
Prefix: [1]	$NP \rightarrow DT NN$	4 [0, $NP \rightarrow DT_{0,1} \bullet NN, 2$]	2, 3, 4	1
			3, 4	2
Prefix: [3]	$VP \rightarrow Vi$	5 [2, $VP \rightarrow Vi_{2,3} \bullet, 3$]	4, 5	3
Complete: [4] [2]		6 [0, $NP \rightarrow DT_{0,1} NN_{1,2} \bullet, 2$]	5, 6	4
			6	5
Prefix: [6]	$S \rightarrow NP VP$	7 [0, $S \rightarrow NP_{0,2} \bullet VP, 2$]	7	6

CKY + Example

$S \rightarrow NP VP$	$Vi \rightarrow \text{sleeps}$
$VP \rightarrow Vi$	$Vt \rightarrow \text{saw}$
$VP \rightarrow Vt NP$	$NN \rightarrow \text{man}$
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$NP \rightarrow NP PP$	$DT \rightarrow \text{the}$
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Input: *the man sleeps*

Rule	Condition	Item	Active	Passive
Axiom	$DT \rightarrow \text{the}$	1 [0, $DT \rightarrow \text{the} \bullet, 1$]	1	
	$NN \rightarrow \text{man}$	2 [1, $NN \rightarrow \text{man} \bullet, 2$]	1, 2	
	$Vi \rightarrow \text{sleeps}$	3 [2, $Vi \rightarrow \text{sleeps} \bullet, 3$]	1, 2, 3	
Prefix: [1]	$NP \rightarrow DT NN$	4 [0, $NP \rightarrow DT_{0,1} \bullet NN, 2$]	2, 3, 4	1
			3, 4	2
Prefix: [3]	$VP \rightarrow Vi$	5 [2, $VP \rightarrow Vi_{2,3} \bullet, 3$]	4, 5	3
Complete: [4] [2]		6 [0, $NP \rightarrow DT_{0,1} NN_{1,2} \bullet, 2$]	5, 6	4
			6	5
Prefix: [6]	$S \rightarrow NP VP$	7 [0, $S \rightarrow NP_{0,2} \bullet VP, 2$]	7	6
Complete: [7] [5]		8 [0, $S \rightarrow NP_{0,2} VP_{2,3} \bullet, 3$]	8	7

CKY + Example

Input: *the man sleeps*

$S \rightarrow NP VP$	$Vi \rightarrow \text{sleeps}$
$VP \rightarrow Vi$	$Vt \rightarrow \text{saw}$
$VP \rightarrow Vt NP$	$NN \rightarrow \text{man}$
$VP \rightarrow VP PP$	$NN \rightarrow \text{dog}$
$NP \rightarrow DT NN$	$NN \rightarrow \text{telescope}$
$NP \rightarrow NP PP$	$DT \rightarrow \text{the}$
$PP \rightarrow IN NP$	$IN \rightarrow \text{with}$

Rule	Condition	Item	Active	Passive
Axiom	$DT \rightarrow \text{the}$	1 [0, $DT \rightarrow \text{the} \bullet$, 1]	1	
	$NN \rightarrow \text{man}$	2 [1, $NN \rightarrow \text{man} \bullet$, 2]	1, 2	
	$Vi \rightarrow \text{sleeps}$	3 [2, $Vi \rightarrow \text{sleeps} \bullet$, 3]	1, 2, 3	
Prefix: [1]	$NP \rightarrow DT NN$	4 [0, $NP \rightarrow DT_{0,1} \bullet NN$, 2]	2, 3, 4	1
			3, 4	2
Prefix: [3]	$VP \rightarrow Vi$	5 [2, $VP \rightarrow Vi_{2,3} \bullet$, 3]	4, 5	3
Complete: [4] [2]		6 [0, $NP \rightarrow DT_{0,1} NN_{1,2} \bullet$, 2]	5, 6	4
			6	5
Prefix: [6]	$S \rightarrow NP VP$	7 [0, $S \rightarrow NP_{0,2} \bullet VP$, 2]	7	6
Complete: [7] [5]		8 [0, $S \rightarrow NP_{0,2} VP_{2,3} \bullet$, 3]	8	7
GOAL: [8]			\emptyset	

Correctness of Parsing Strategy

Soundness: if a goal item is proven for **s**

- then **s** \in L(G)

Completeness: if **s** \in L(G)

- then a goal item can be proven for **s**

Parse Forest

Efficient representation of the whole space $T_G(\mathbf{s})$

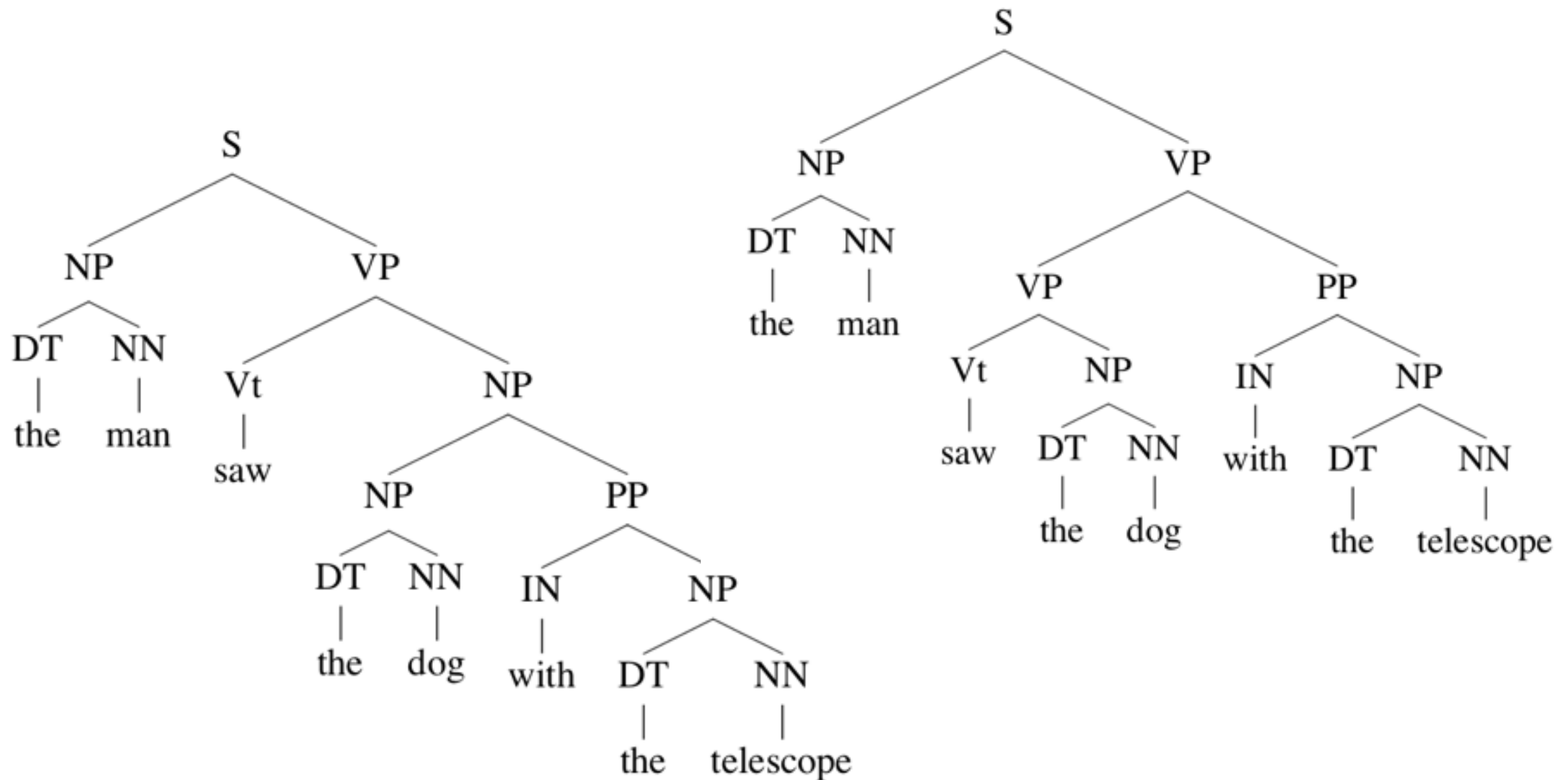
- each and every possible tree yielding \mathbf{s}

We must be able to represent partial derivations

- including alternative ones

Ambiguity

Some strings may have more than one derivation in G



Dealing with Ambiguity

Statistical model: weight steps in a derivation

- induces a partial ordering over derivations
- can be used to make a decision
 - e.g. best tree under the model

Probabilistic CFG

CFG extended with parameters $0 \leq \theta_r \leq 1$

- where $r \in R$ and

$$\sum_{\alpha: X \rightarrow \alpha \in R} \theta_{X \rightarrow \alpha} = 1$$

Probabilistic CFG

Distribution over trees

$$\begin{aligned} P(T = t, S = \text{yield}(t)) &= P(T = \langle r_1 \dots r_n \rangle, S = s) \\ &= \prod_{i=1}^n \theta_{r_i} = \prod_{i=1}^n \theta_{X_i \rightarrow \alpha_i} = \prod_{r \in t} \theta_r^{n(r,t)} \end{aligned}$$

and strings

$$P(S = s) = \sum_{t \in T_G(s)} P(T = t, S = s)$$

Estimation

Let us assume the parametric form of θ is a multinomial

- one categorical distribution per $X \in N$

Suppose we can observe a *treebank*, then by MLE

$$\begin{aligned}\theta_{X \rightarrow \alpha} &= \frac{n(X \rightarrow \alpha)}{n(X)} \\ &= \frac{n(X \rightarrow \alpha)}{\sum_{\alpha'} n(X \rightarrow \alpha')}\end{aligned}$$

Weighted CKY+

Input: G and $s = w_1 \dots w_n$

Item form: $[i, X \rightarrow \alpha_{\blacksquare} \bullet \beta_{\square}, j]$
asserts that $X \Rightarrow^* w_{i+1} \dots w_j \beta$

Axioms: $[i, X \rightarrow w_i \bullet \alpha_{\square}, i+1] : \theta_r$ $r = X \rightarrow w_i \alpha \in R$
 $[i, X \rightarrow \varepsilon \bullet, i] : \theta_r$ $r = X \rightarrow \varepsilon \in R$

Goal: $[0, S \rightarrow \alpha_{\blacksquare} \bullet, n]$

Scan

$$\frac{[i, X \rightarrow \alpha_{\blacksquare} \bullet w_{j+1} \beta_{\square}, j] : \theta_1}{[i, X \rightarrow \alpha_{\blacksquare} w_{j+1} \bullet \beta_{\square}, j+1] : \theta_1}$$

Prefix

$$\frac{[i, Y \rightarrow \alpha_{\blacksquare} \bullet, j] : \theta_1}{[i, X \rightarrow Y_{i,j} \bullet \beta_{\square}, j] : \theta_r} \quad r = X \rightarrow Y \beta \in R$$

Complete

$$\frac{[i, X \rightarrow \alpha_{\blacksquare} \bullet Y \beta_{\square}, k] : \theta_1 \quad [k, Y \rightarrow \gamma_{\blacksquare} \bullet, j] : \theta_2}{[i, X \rightarrow \alpha_{\blacksquare} Y_{k,j} \bullet \beta_{\square}, j] : \theta_1}$$

Joint Distribution

${}_0S_3 \rightarrow {}_0NP_2 {}_2VP_3$

${}_0NP_2 \rightarrow {}_0DT_1 {}_1NN_2$

${}_2VP_3 \rightarrow {}_2Vi_3$

${}_0DT_1 \rightarrow \text{the}$

${}_1NN_2 \rightarrow \text{man}$

${}_2Vi_3 \rightarrow \text{sleeps}$

Joint Distribution

${}_0S_3 \rightarrow {}_0NP_2 {}_2VP_3$

${}_0NP_2 \rightarrow {}_0DT_1 {}_1NN_2$

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${}_1NN_2 \rightarrow \text{man}$

${}_2Vi_3 \rightarrow \text{sleeps}$

the man sleeps

Joint Distribution

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Joint Distribution

${}_0S_3 \rightarrow {}_0NP_2 {}_2VP_3$

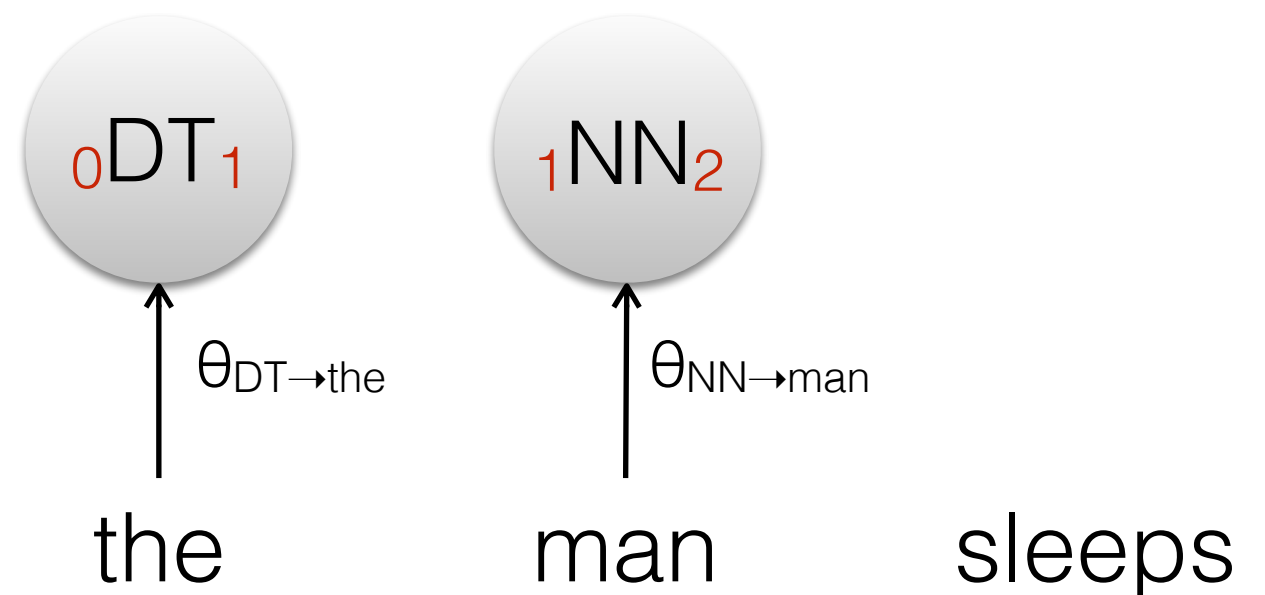
${}_0NP_2 \rightarrow {}_0DT_1 {}_1NN_2$

${}_2VP_3 \rightarrow {}_2Vi_3$

${}_0DT_1 \rightarrow \text{the}$

${}_1NN_2 \rightarrow \text{man}$

${}_2Vi_3 \rightarrow \text{sleeps}$



Joint Distribution

${}_0S_3 \rightarrow {}_0NP_2 {}_2VP_3$

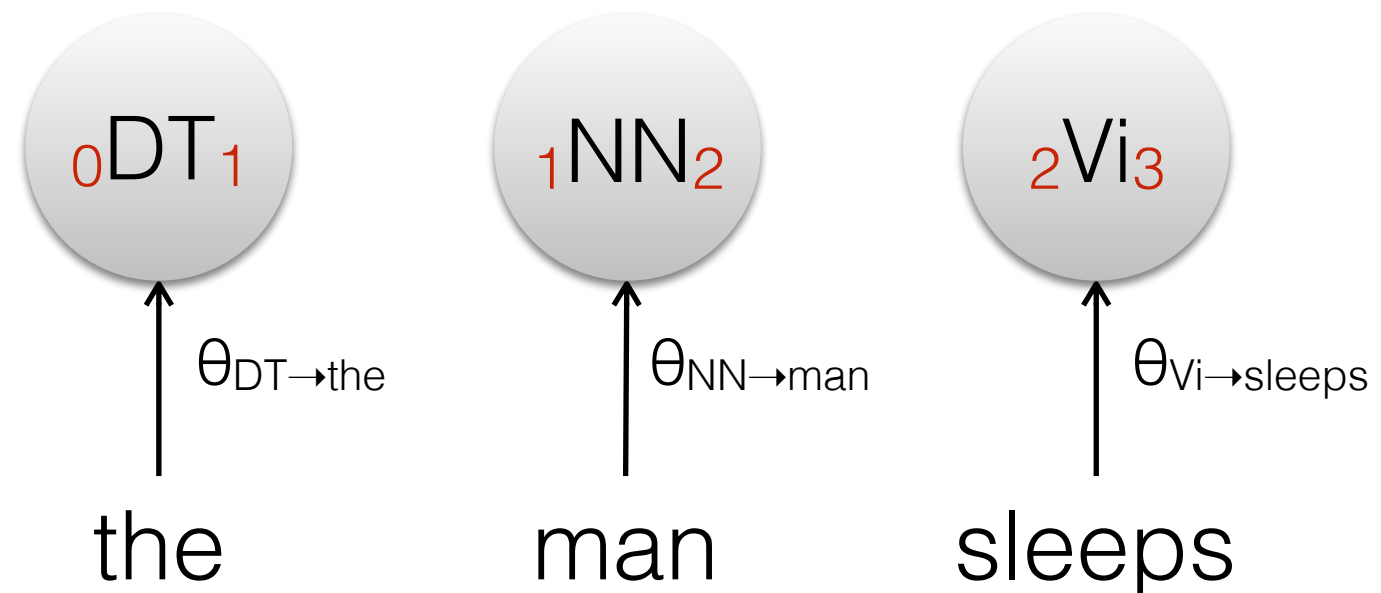
${}_0NP_2 \rightarrow {}_0DT_1 {}_1NN_2$

${}_2VP_3 \rightarrow {}_2Vi_3$

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Joint Distribution

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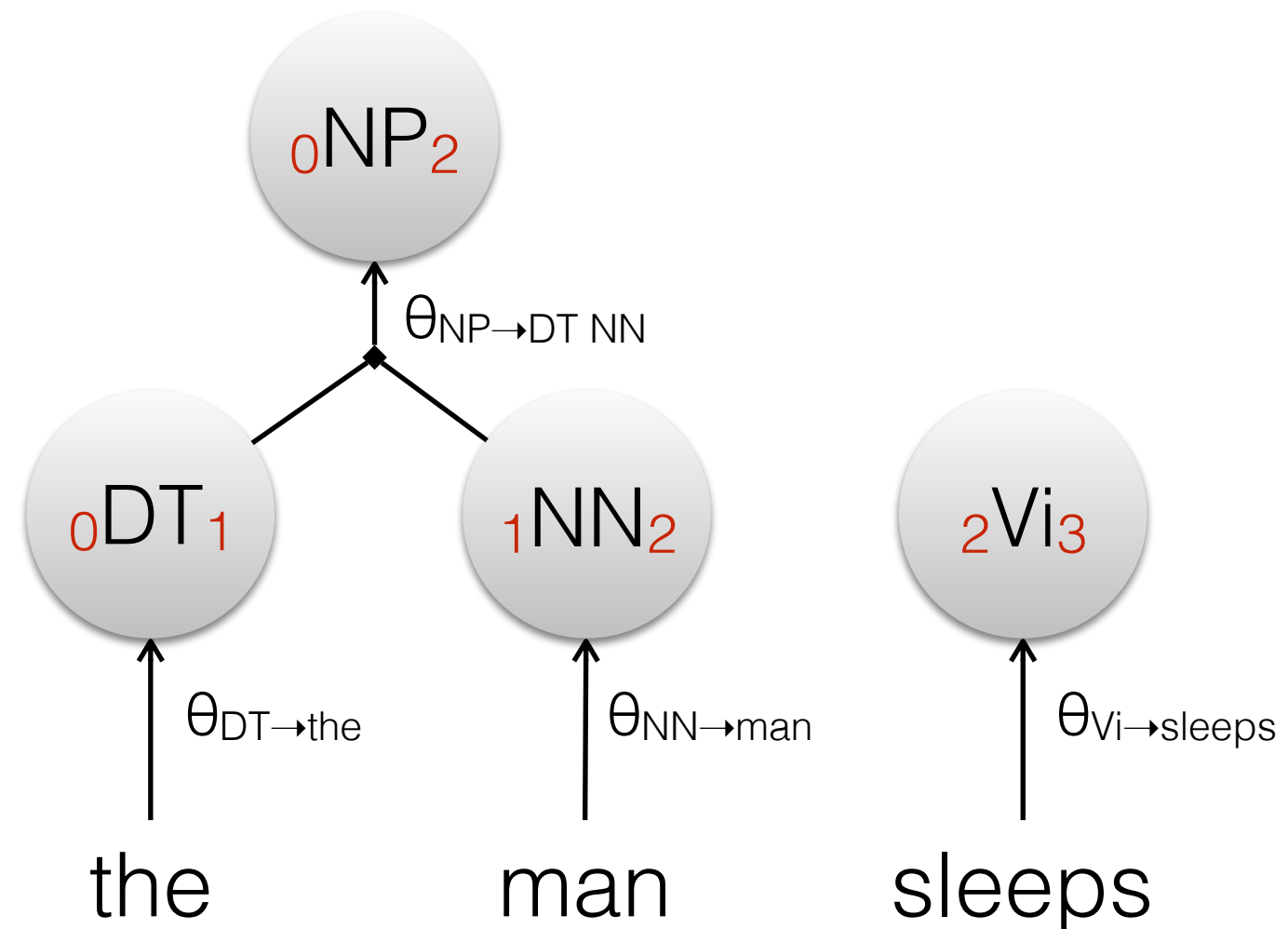
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${}_2VP_3 \rightarrow {}_2Vi_3$

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Joint Distribution

${}^0S_3 \rightarrow {}^0NP_2 {}^2VP_3$

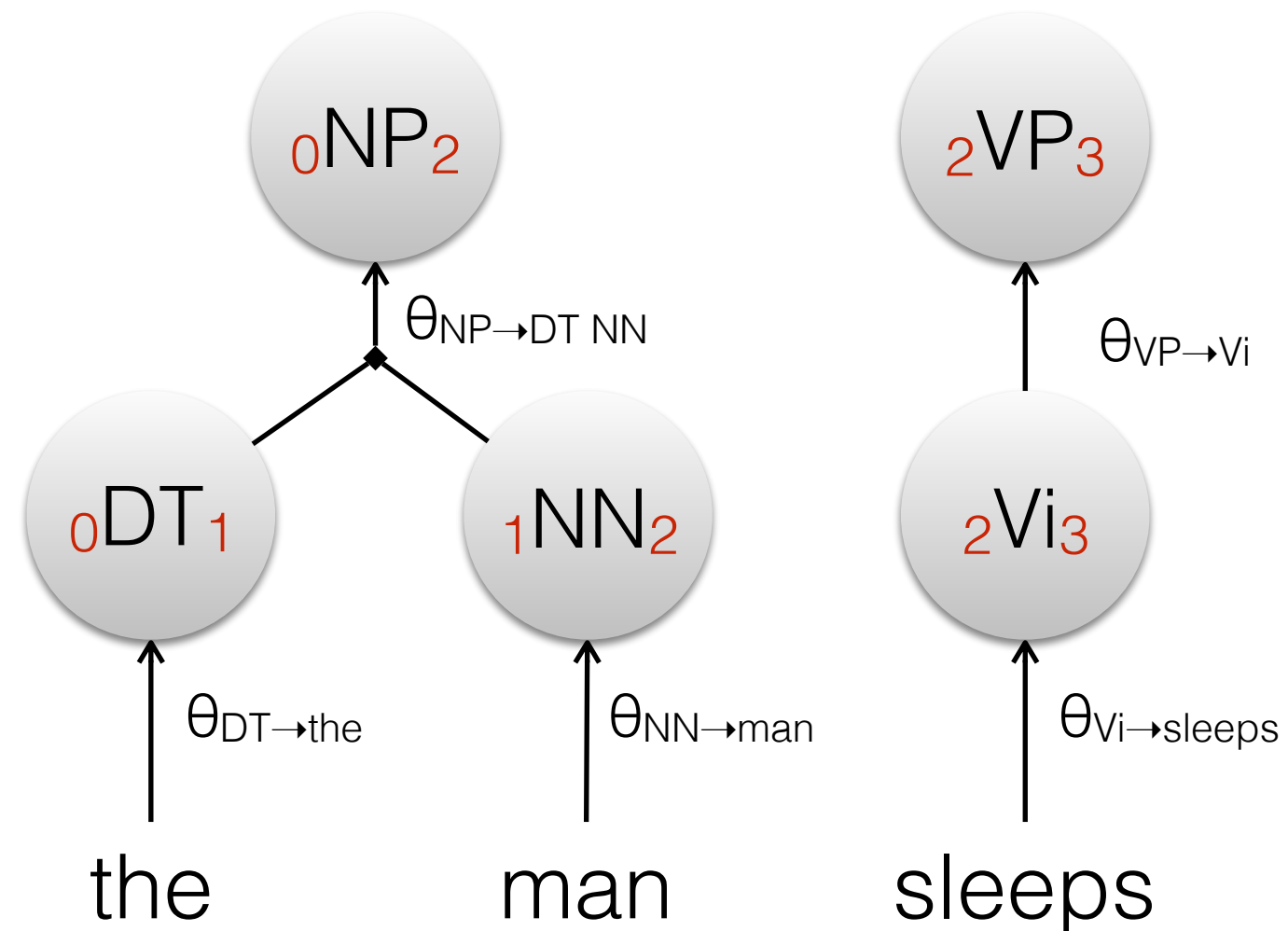
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Joint Distribution

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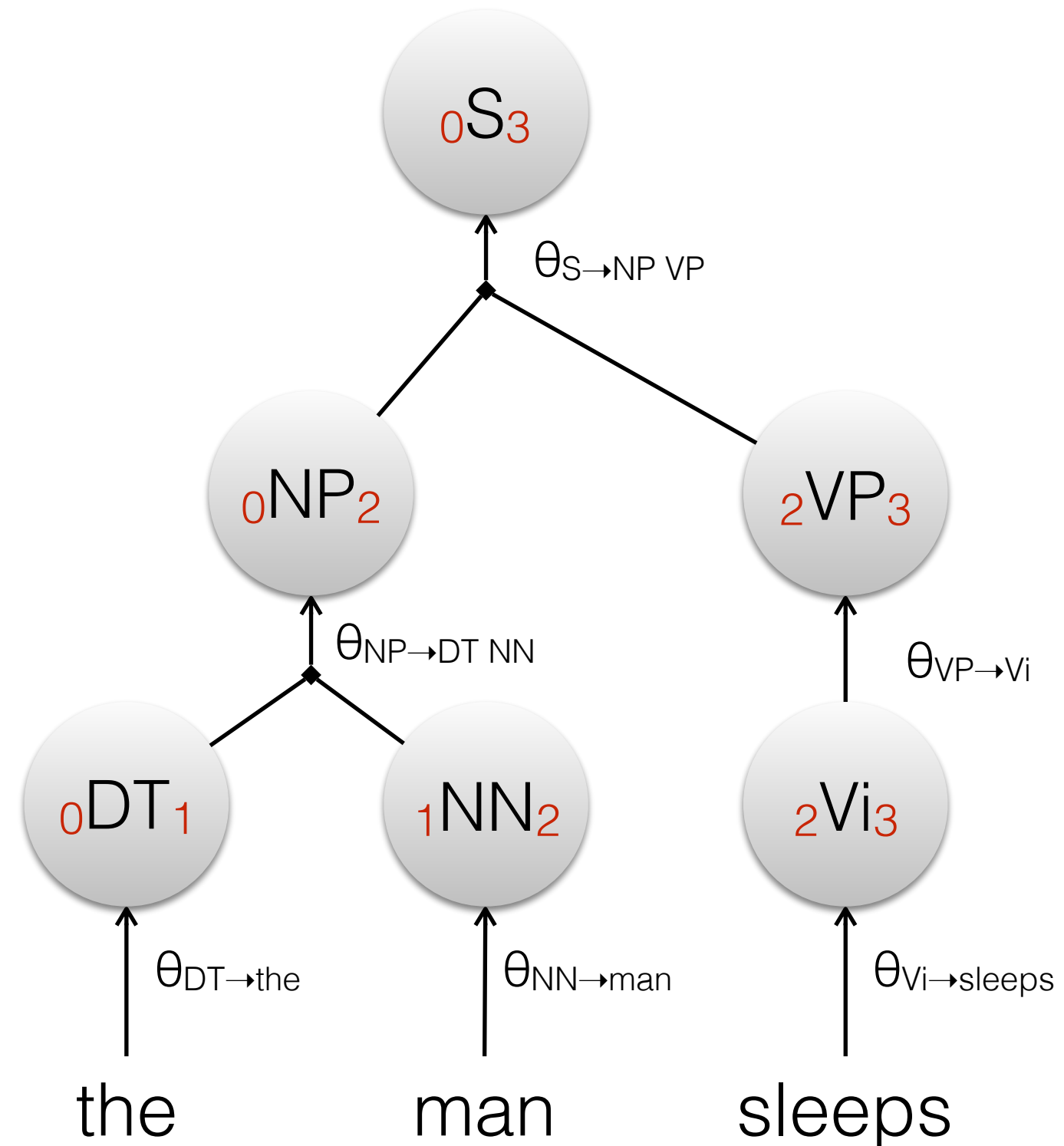
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${}^2VP_3 \rightarrow {}^2Vi_3$

${}^0DT_1 \rightarrow \text{the}$

${}^1NN_2 \rightarrow \text{man}$

${}^2Vi_3 \rightarrow \text{sleeps}$



Ambiguity

Ambiguity

the man

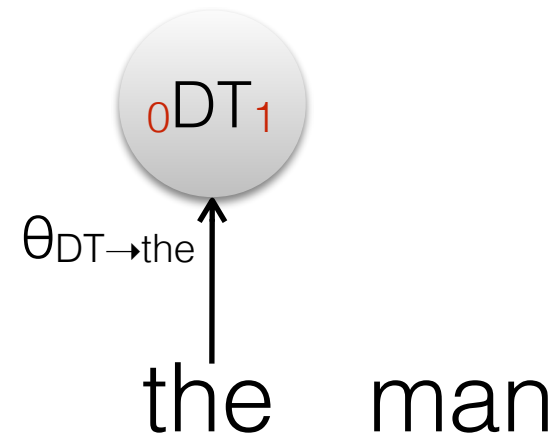
saw

the dog

32

with the telescope

Ambiguity

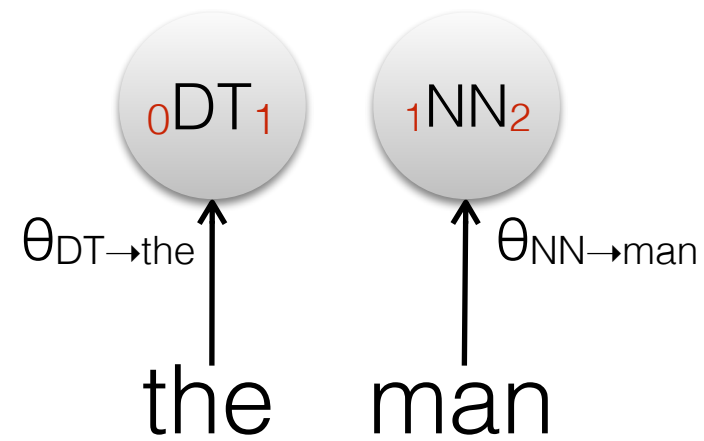


saw

the dog

with the telescope

Ambiguity

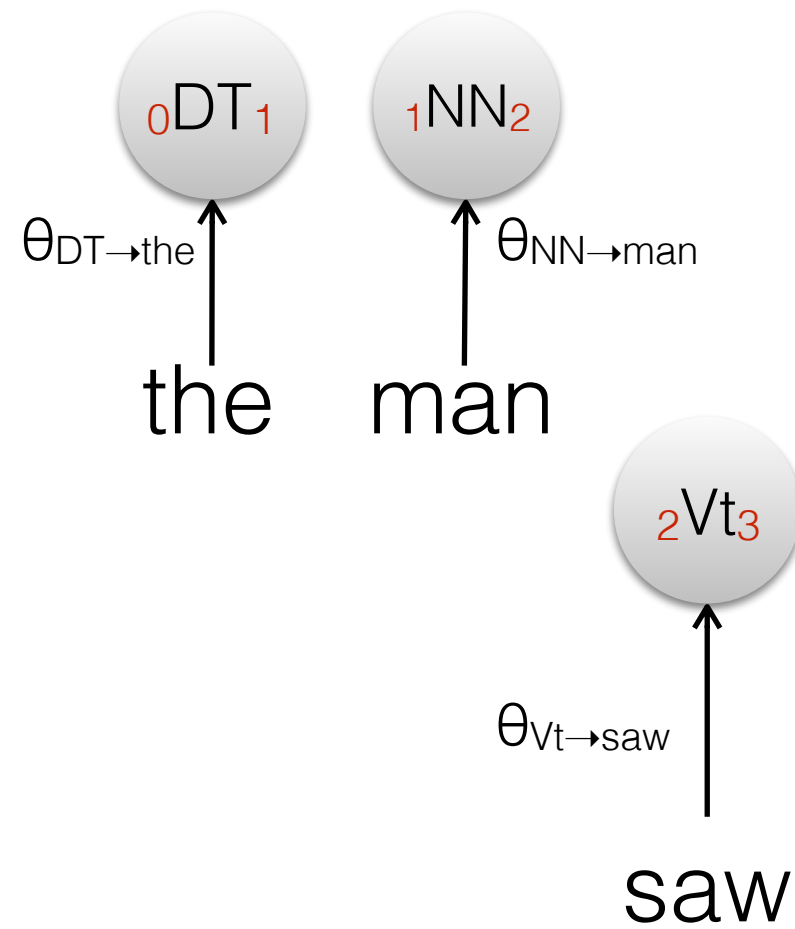


saw

the dog

with the telescope

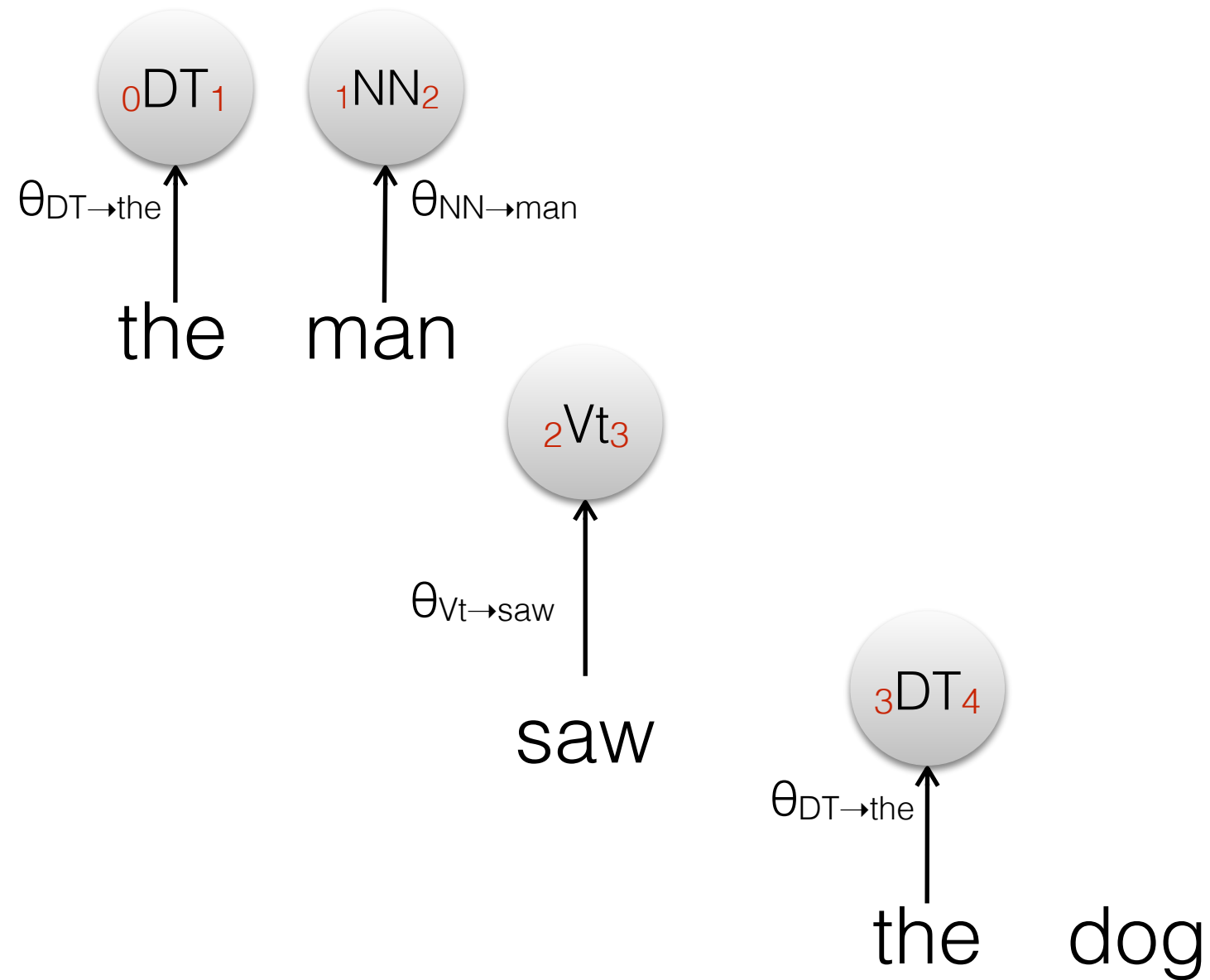
Ambiguity



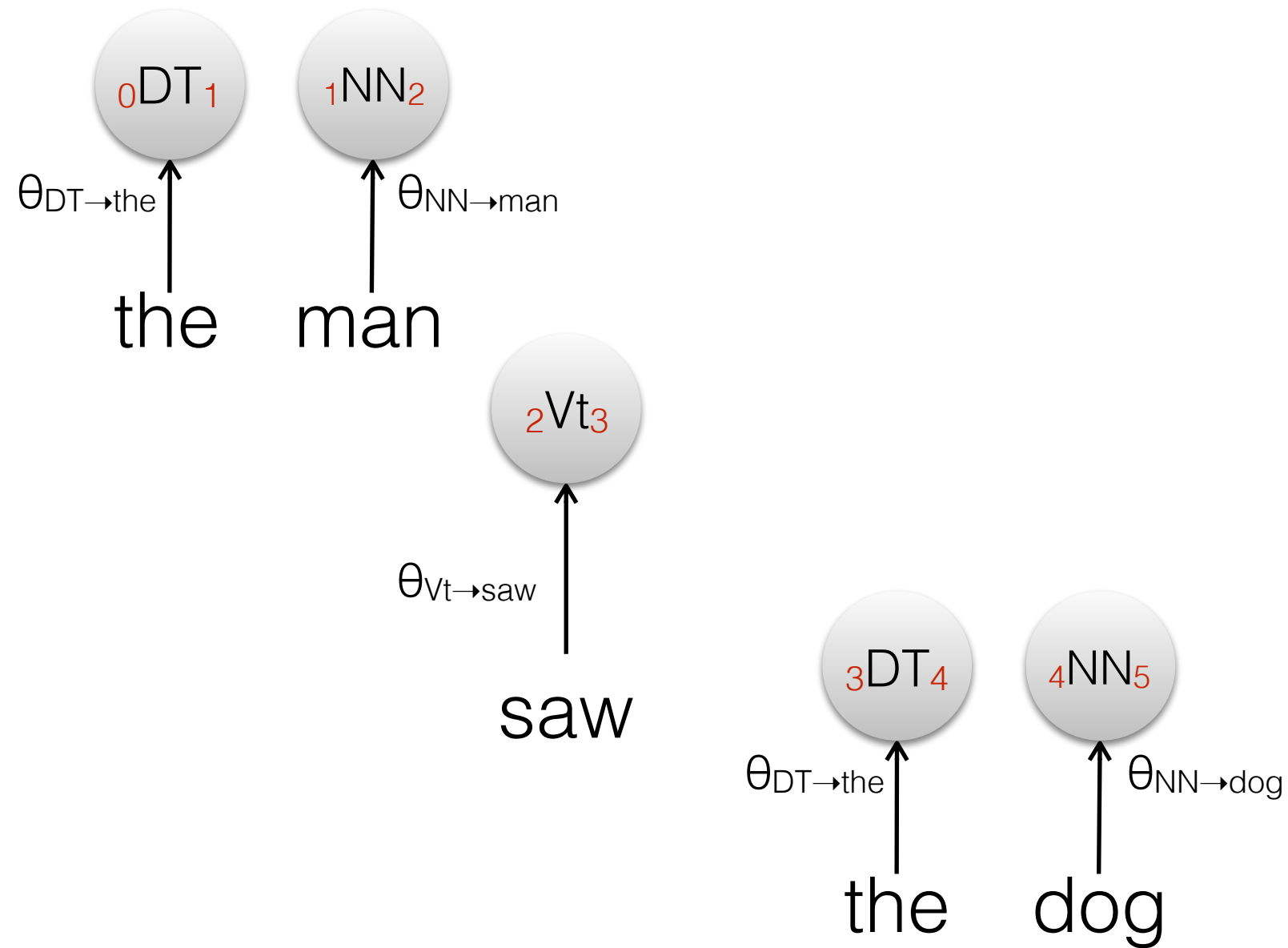
the dog

with the telescope

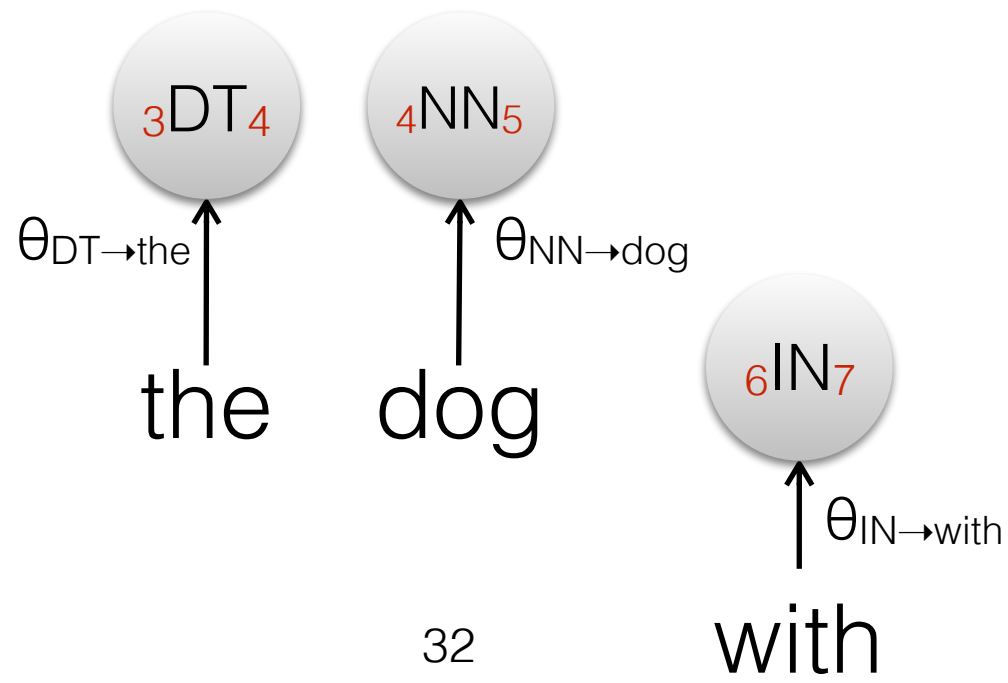
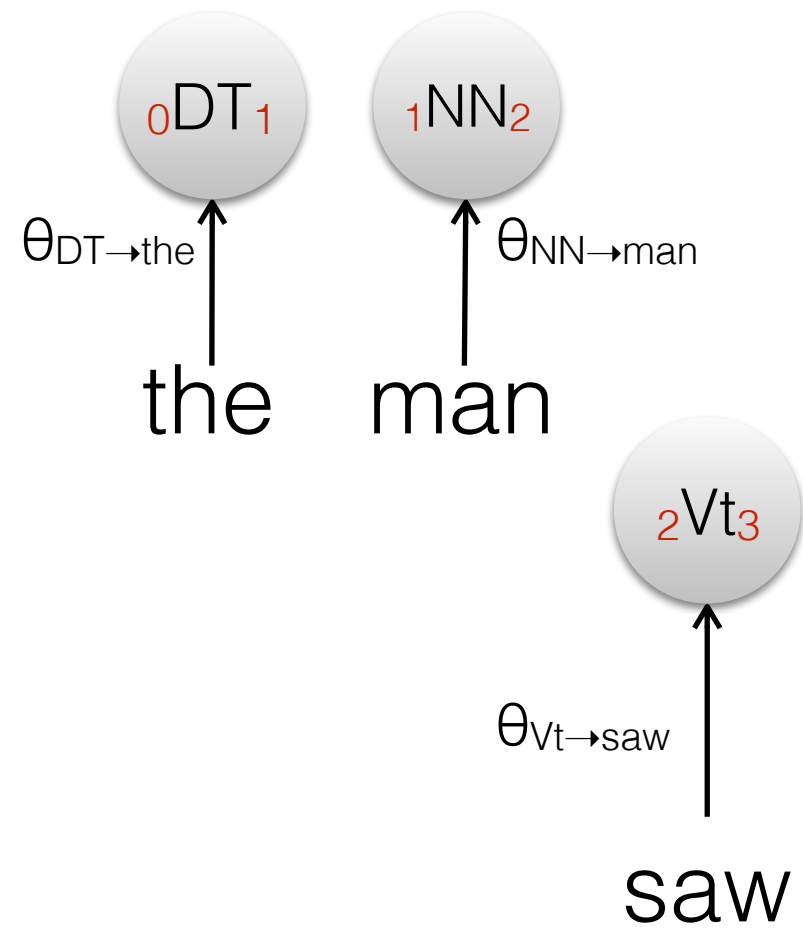
Ambiguity



Ambiguity



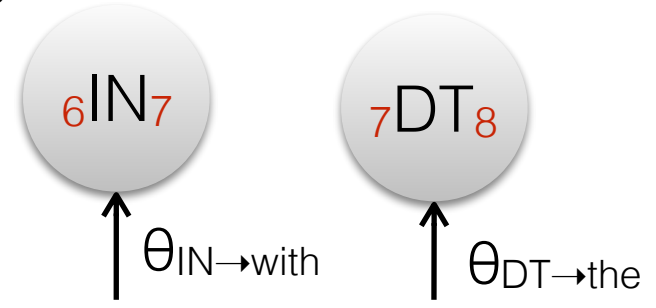
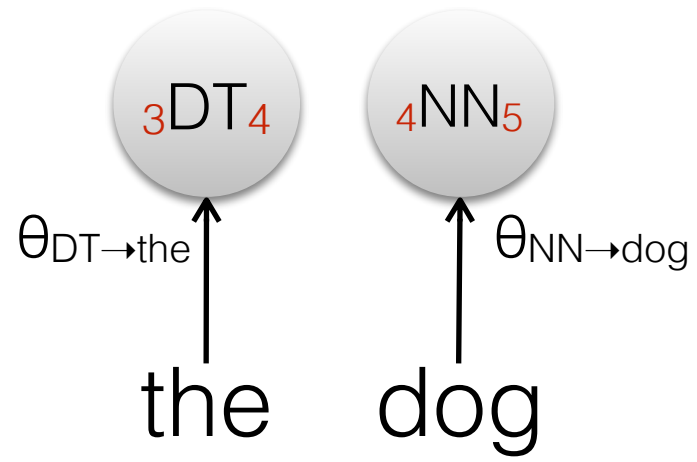
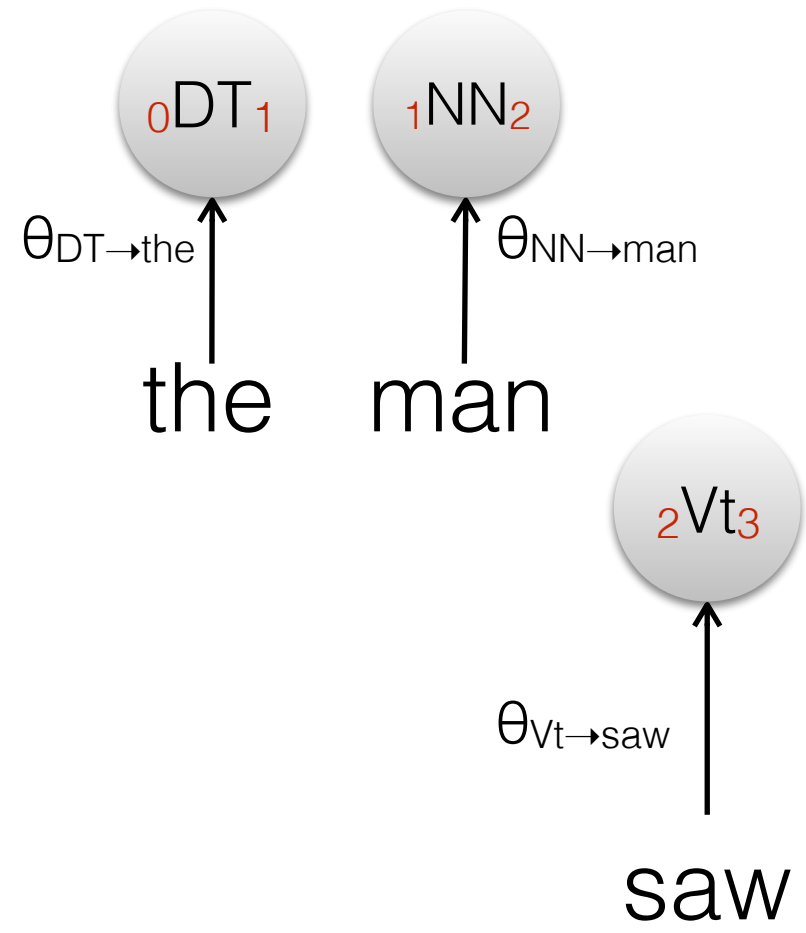
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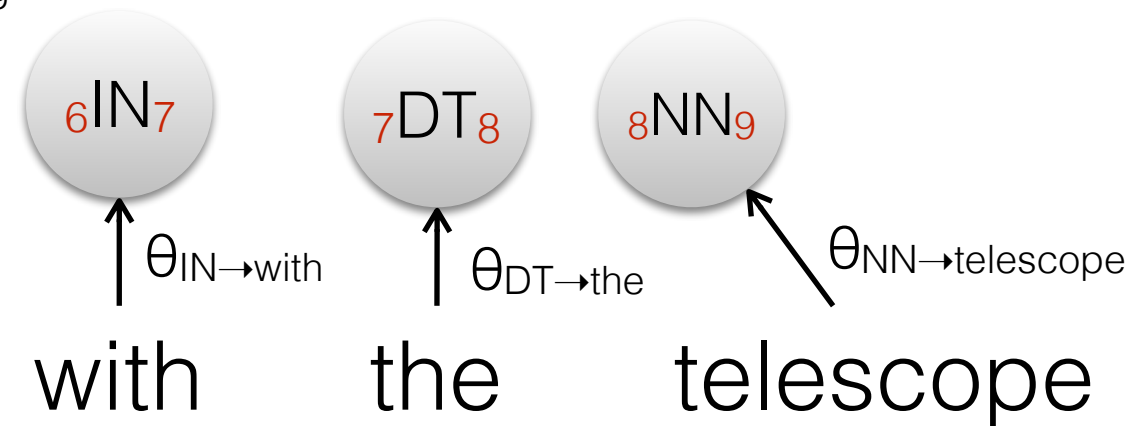
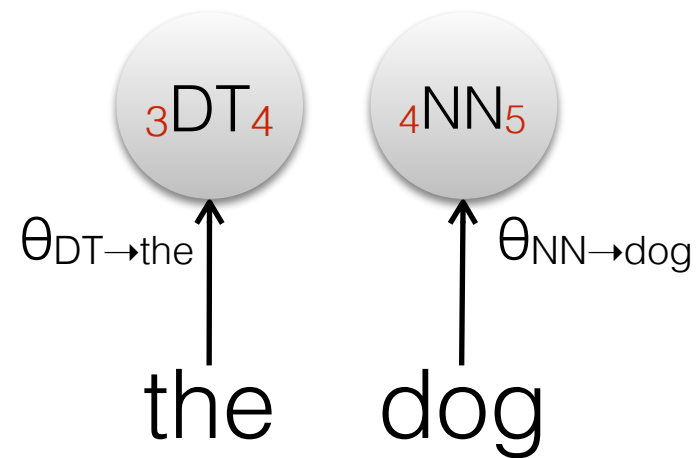
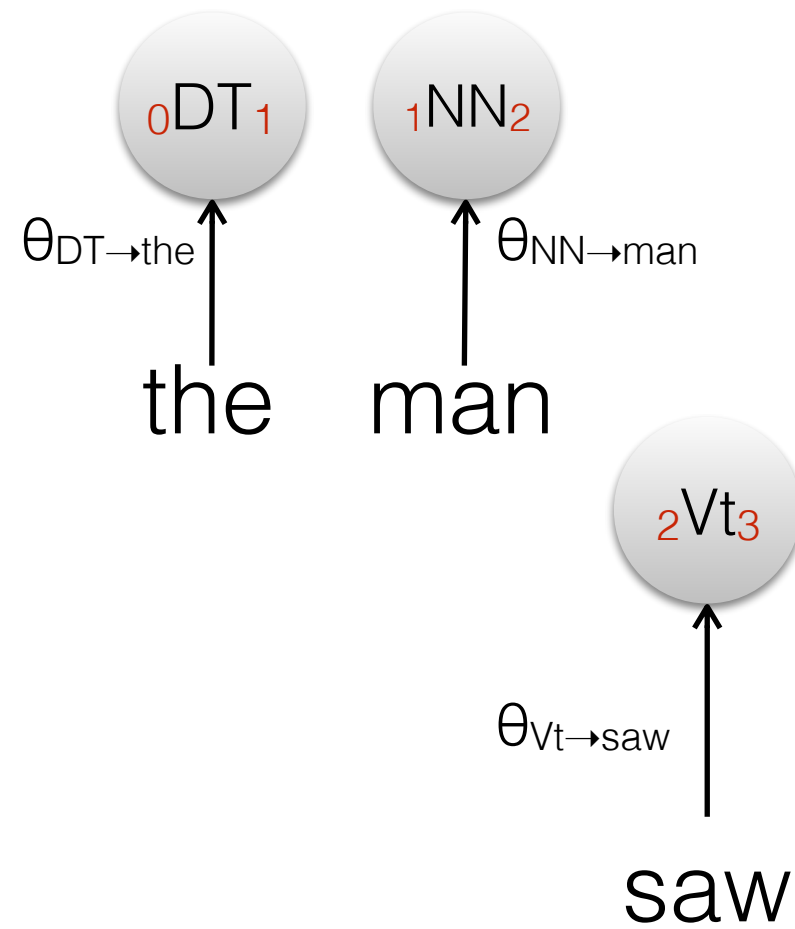
32

with the telescope

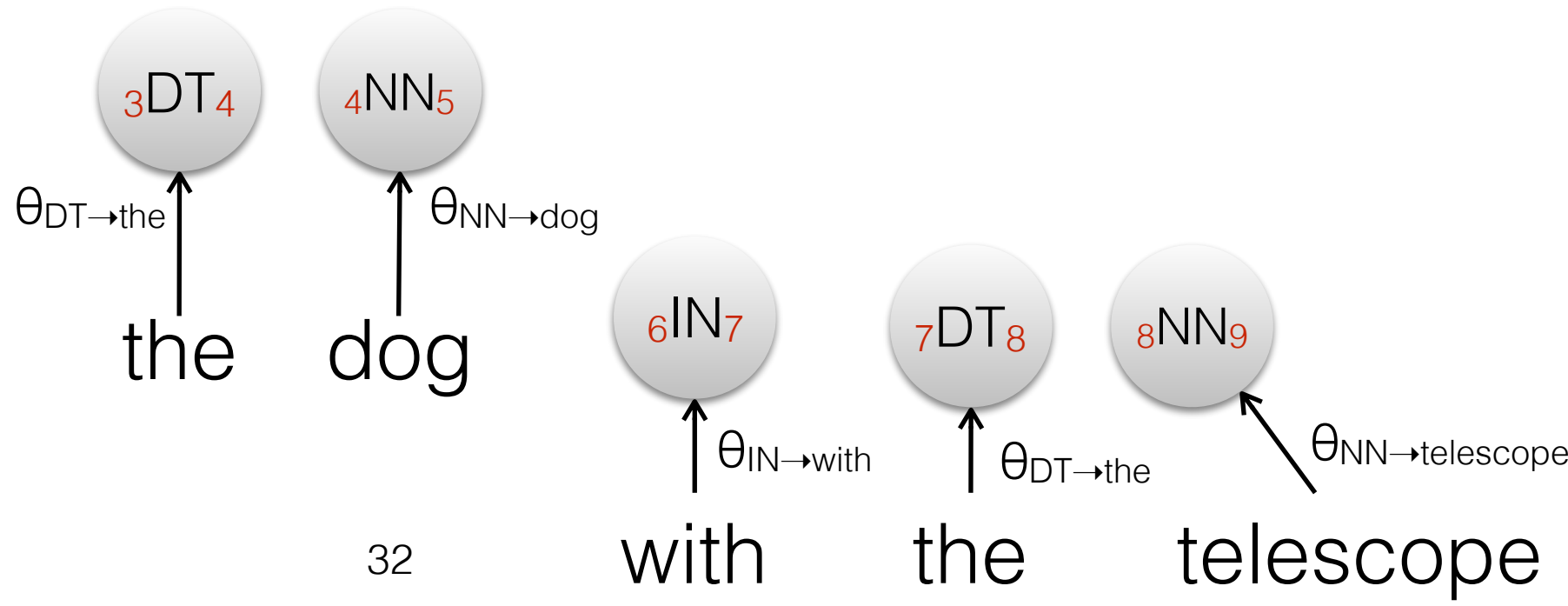
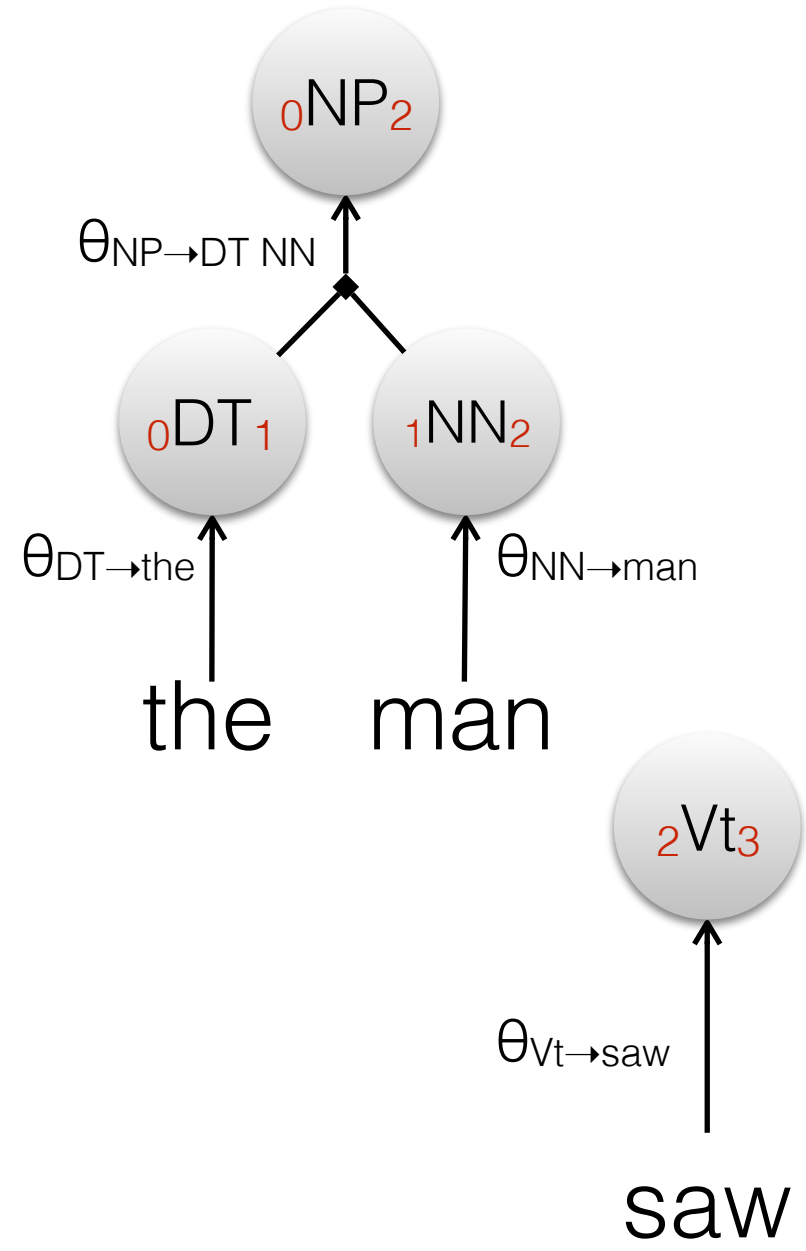
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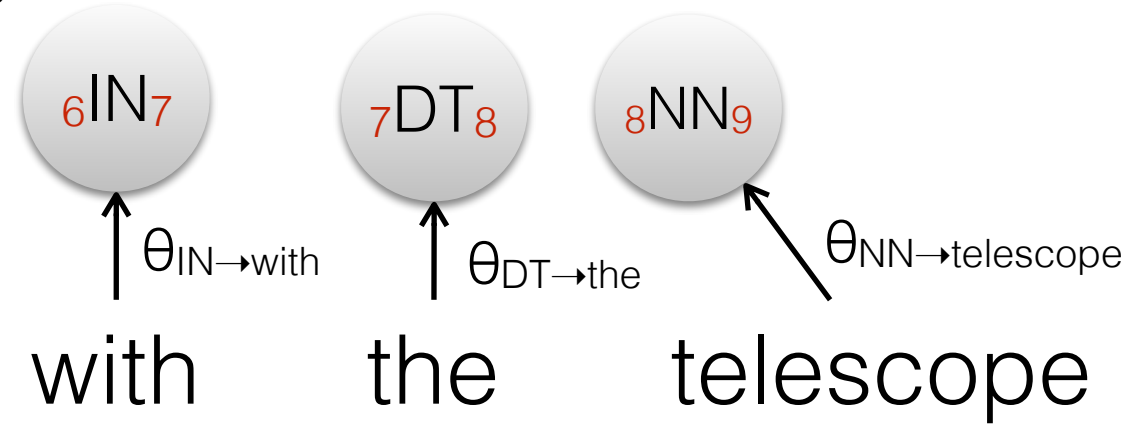
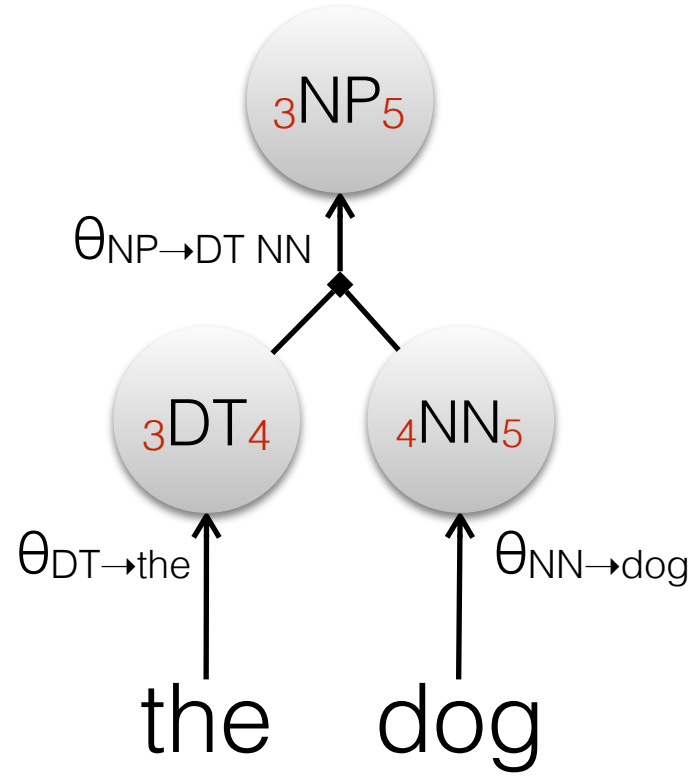
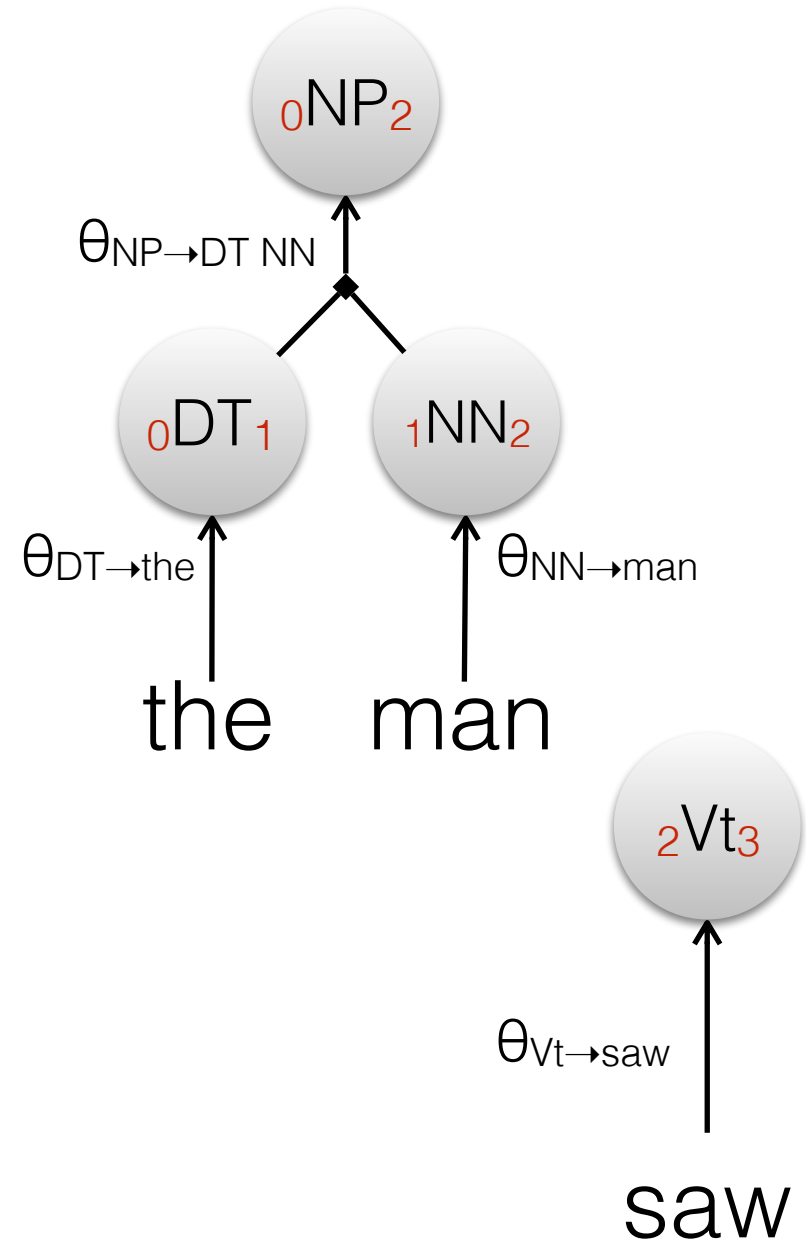
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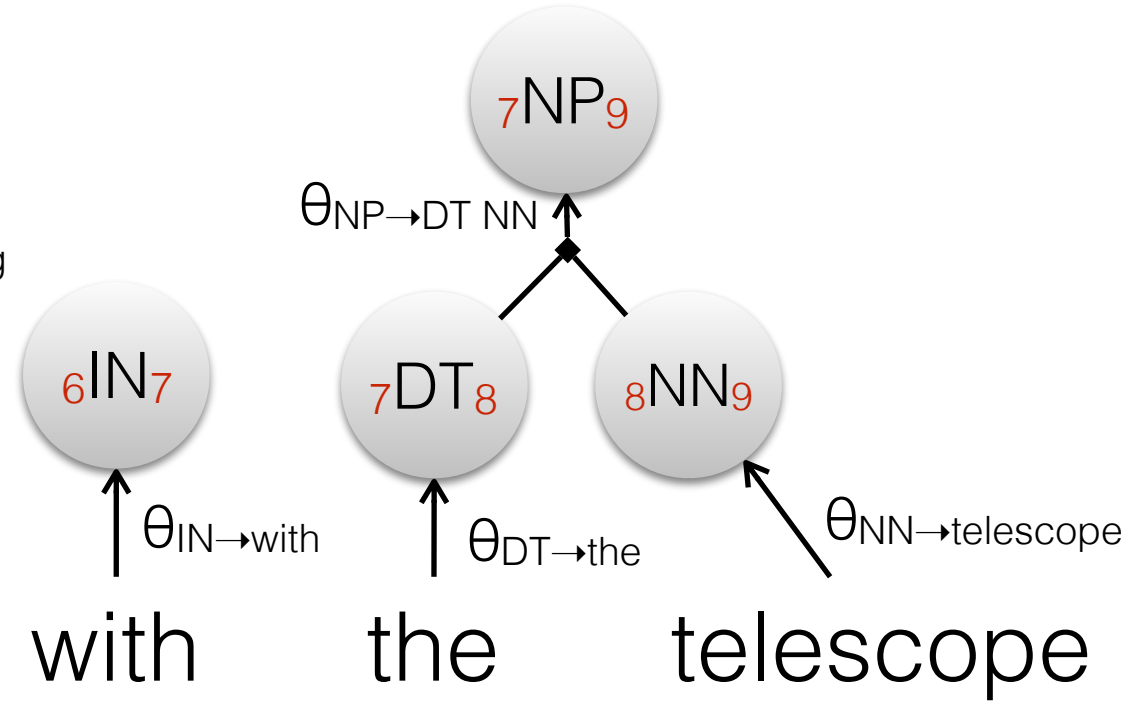
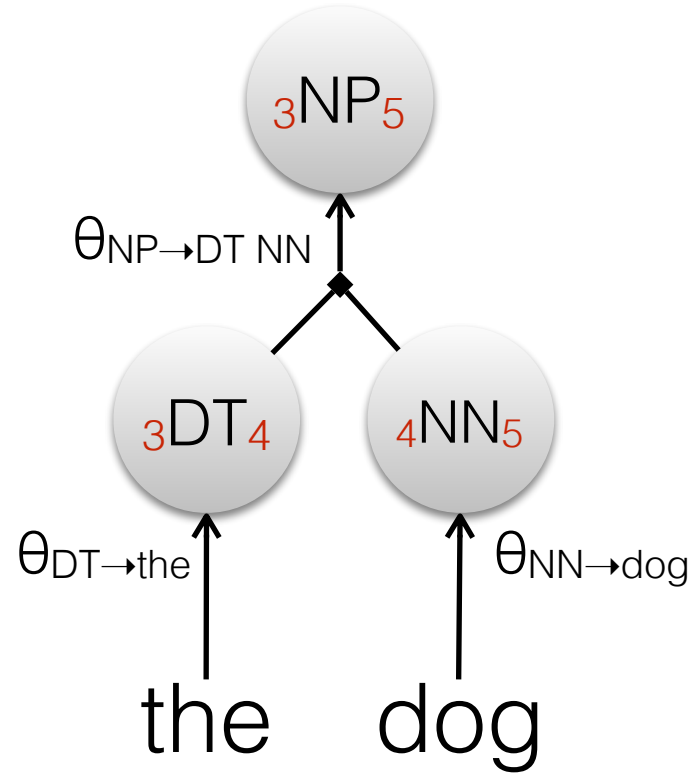
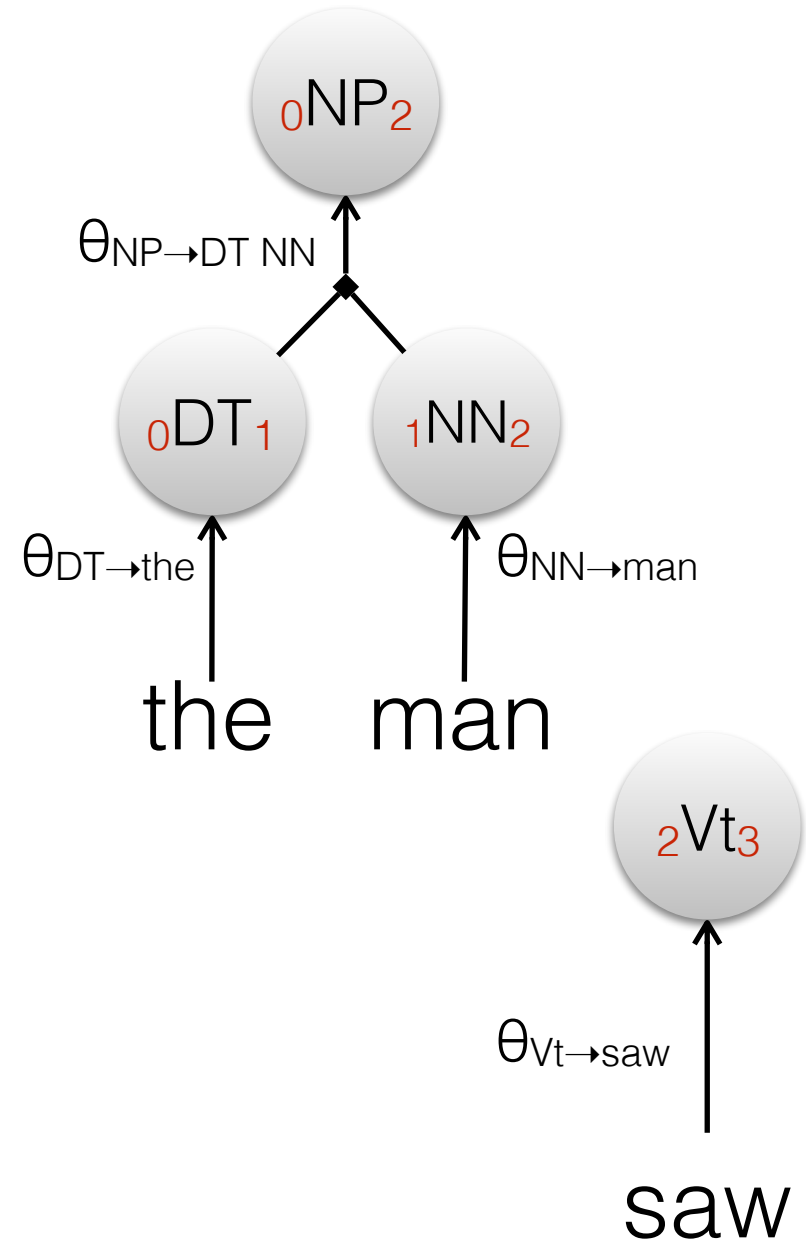
Ambiguity



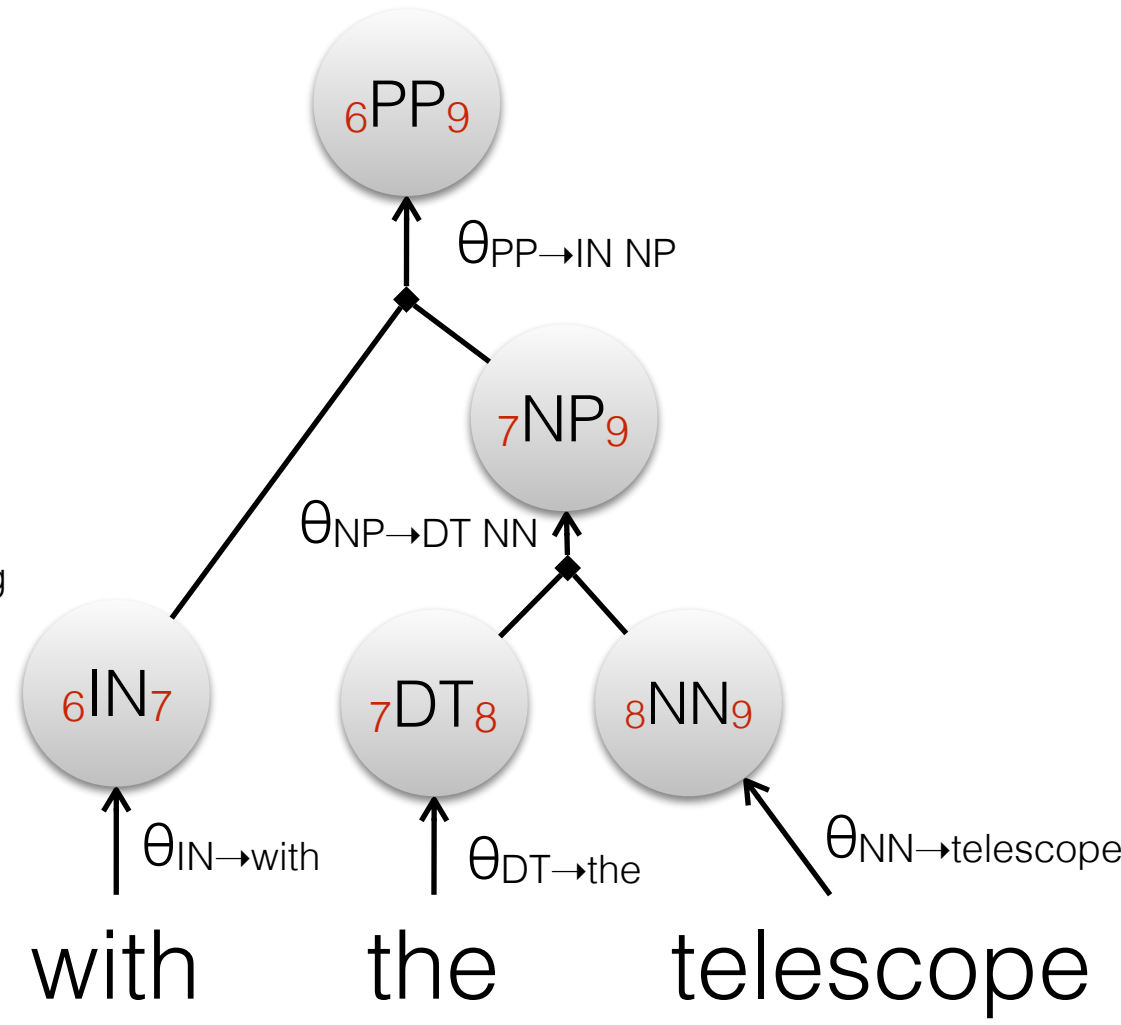
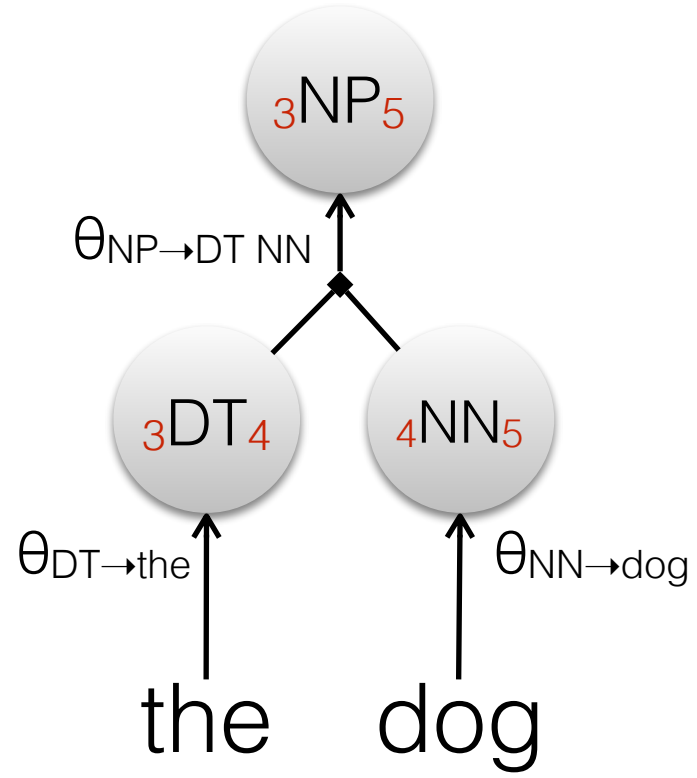
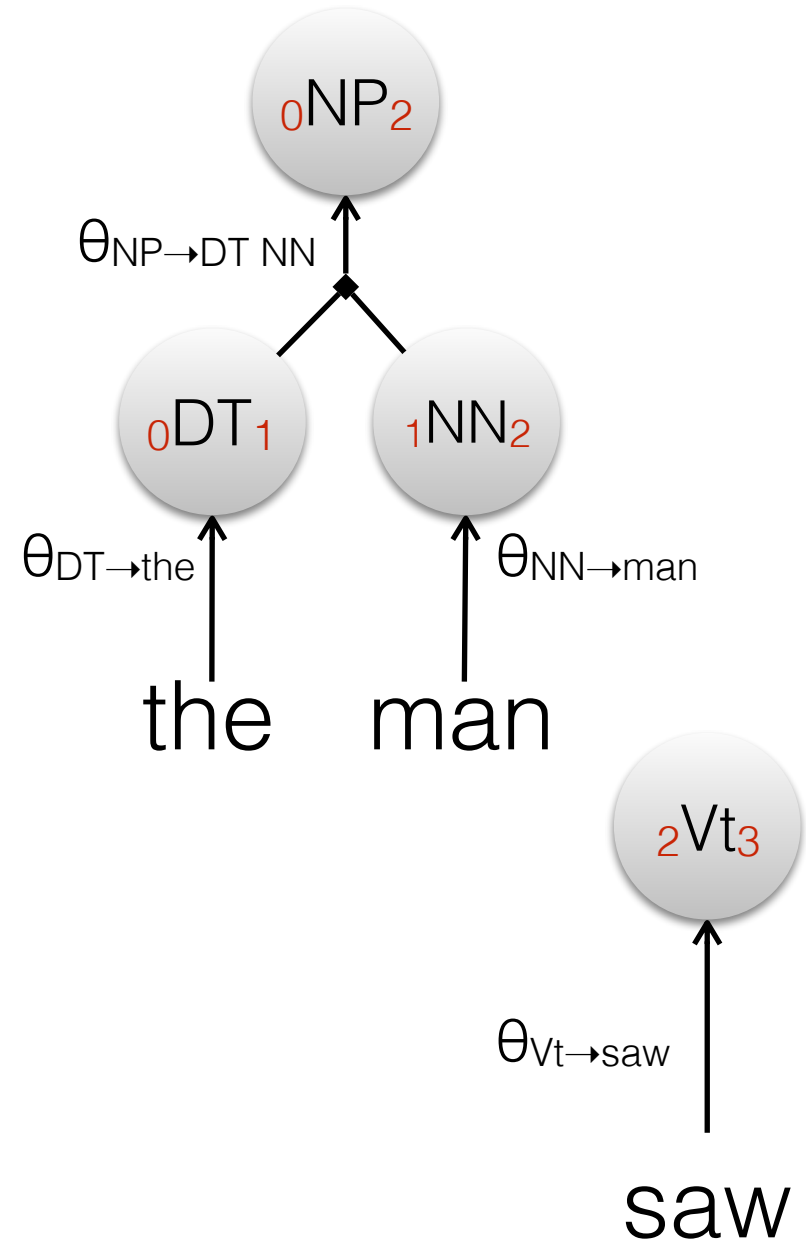
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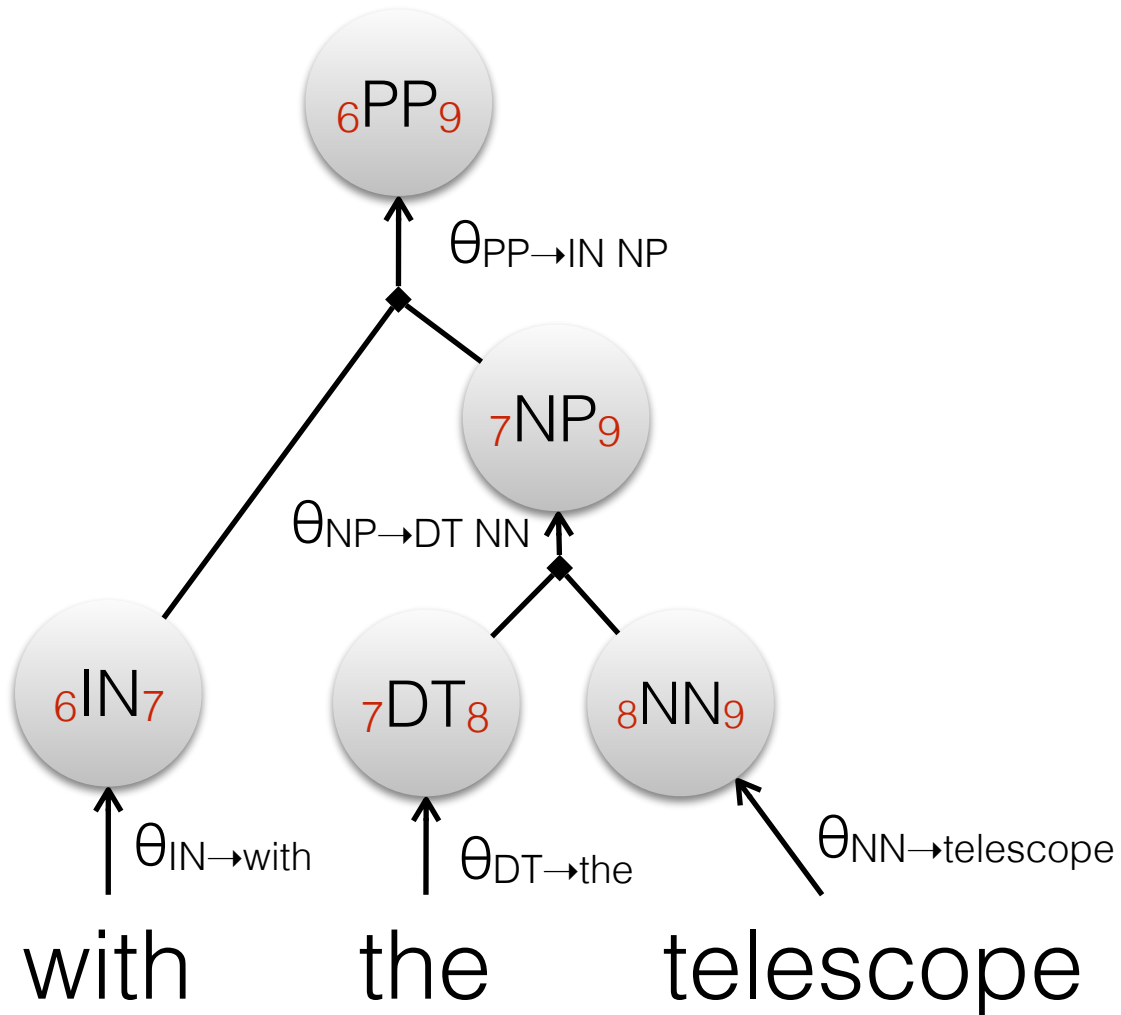
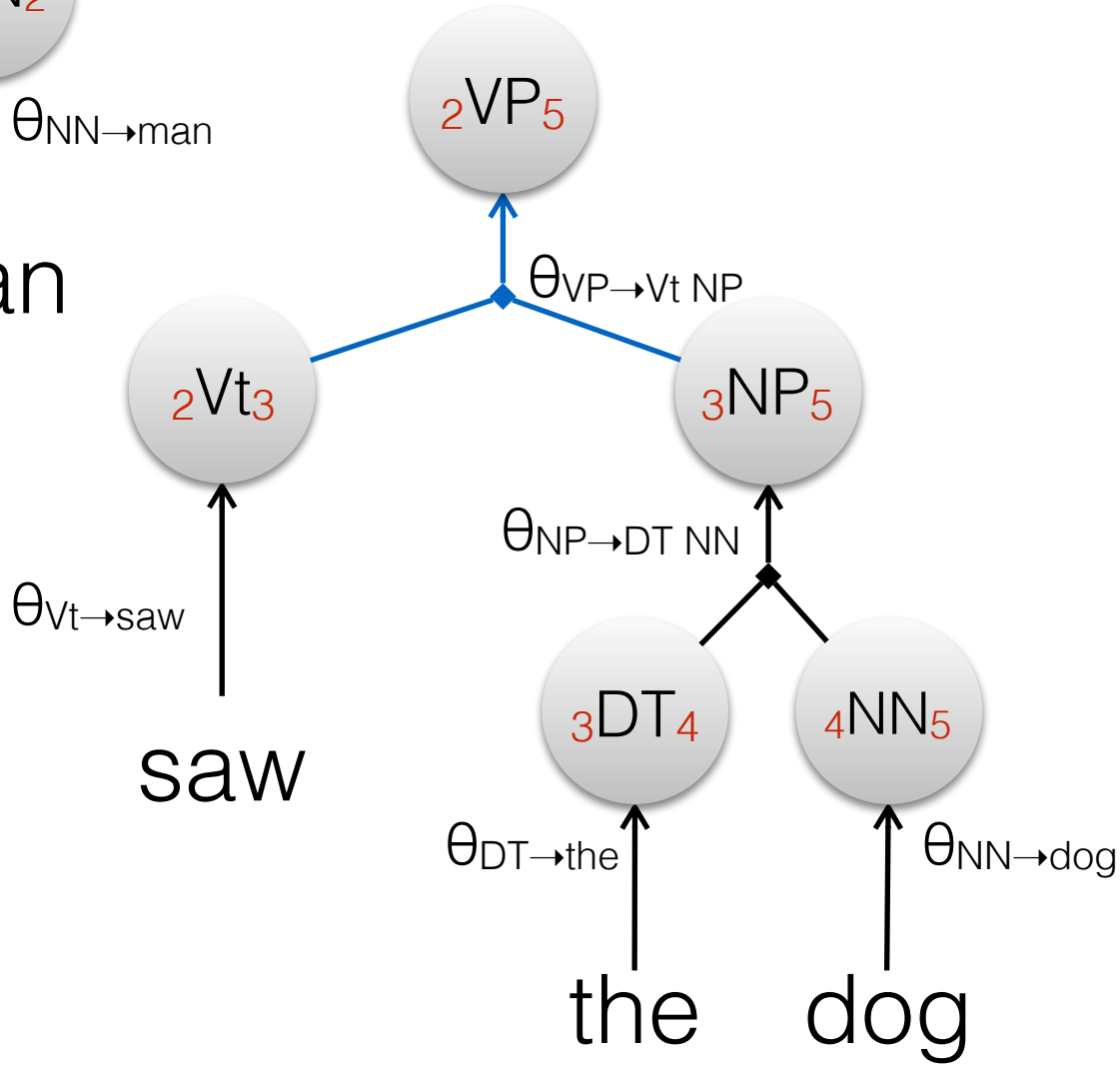
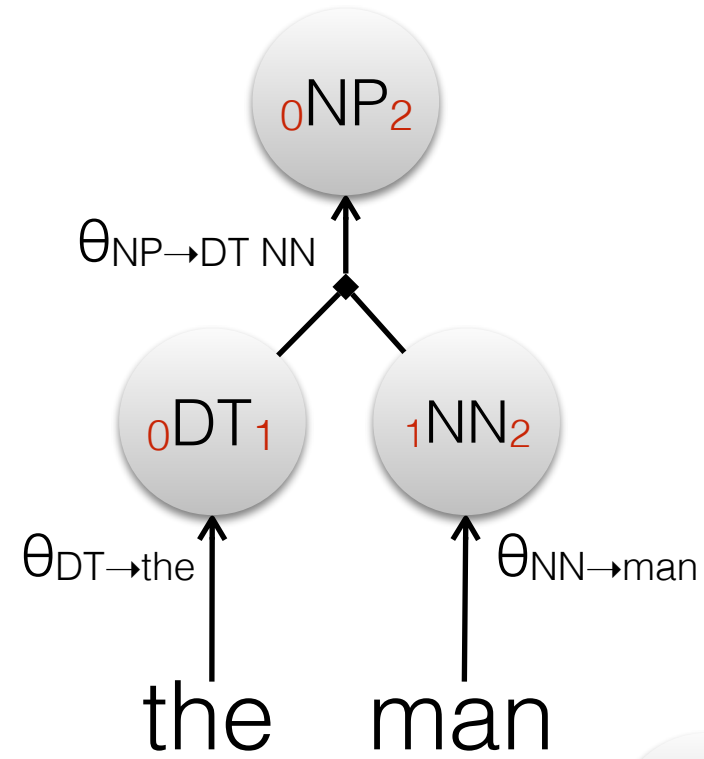
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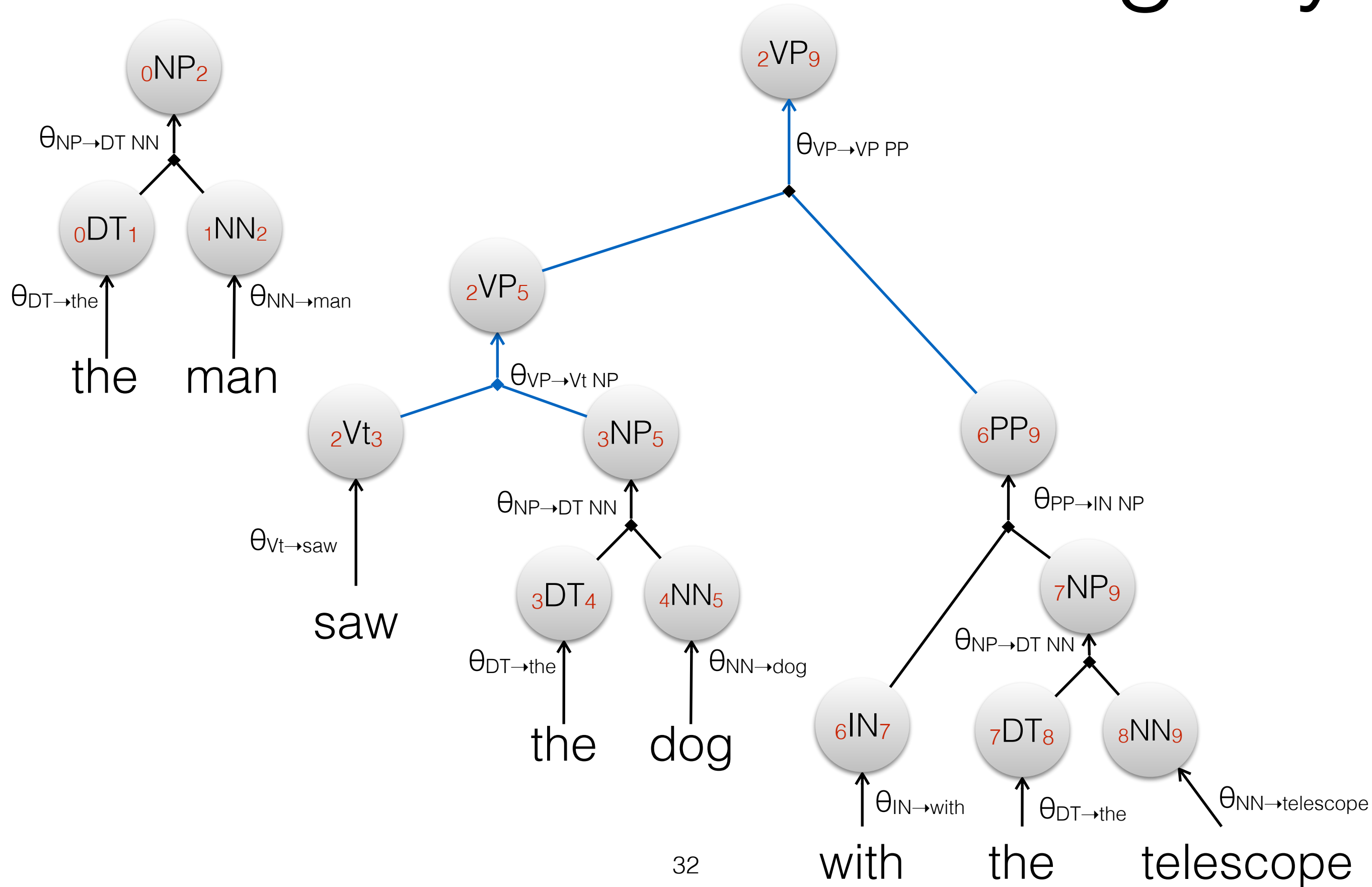
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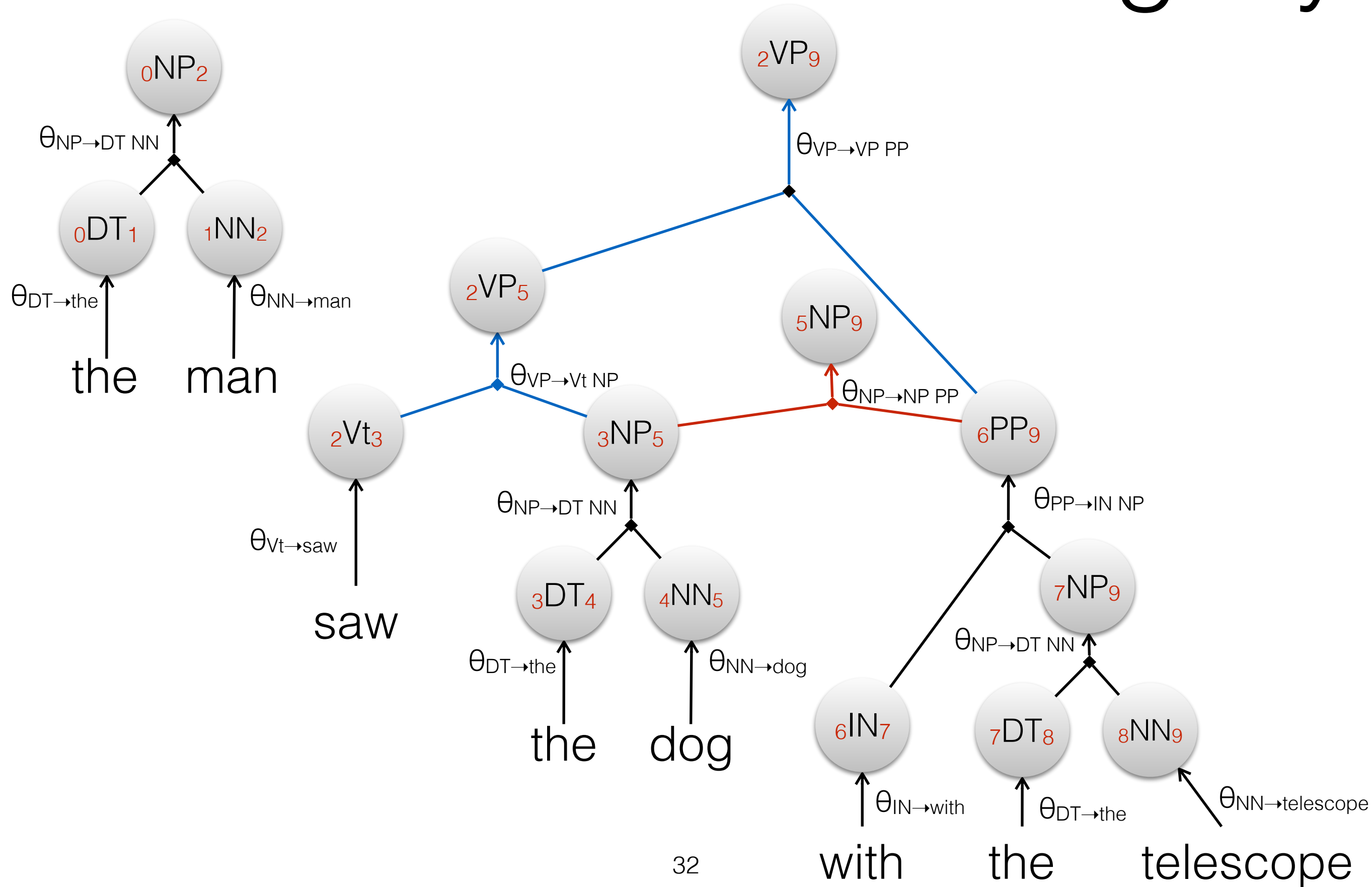
Ambiguity



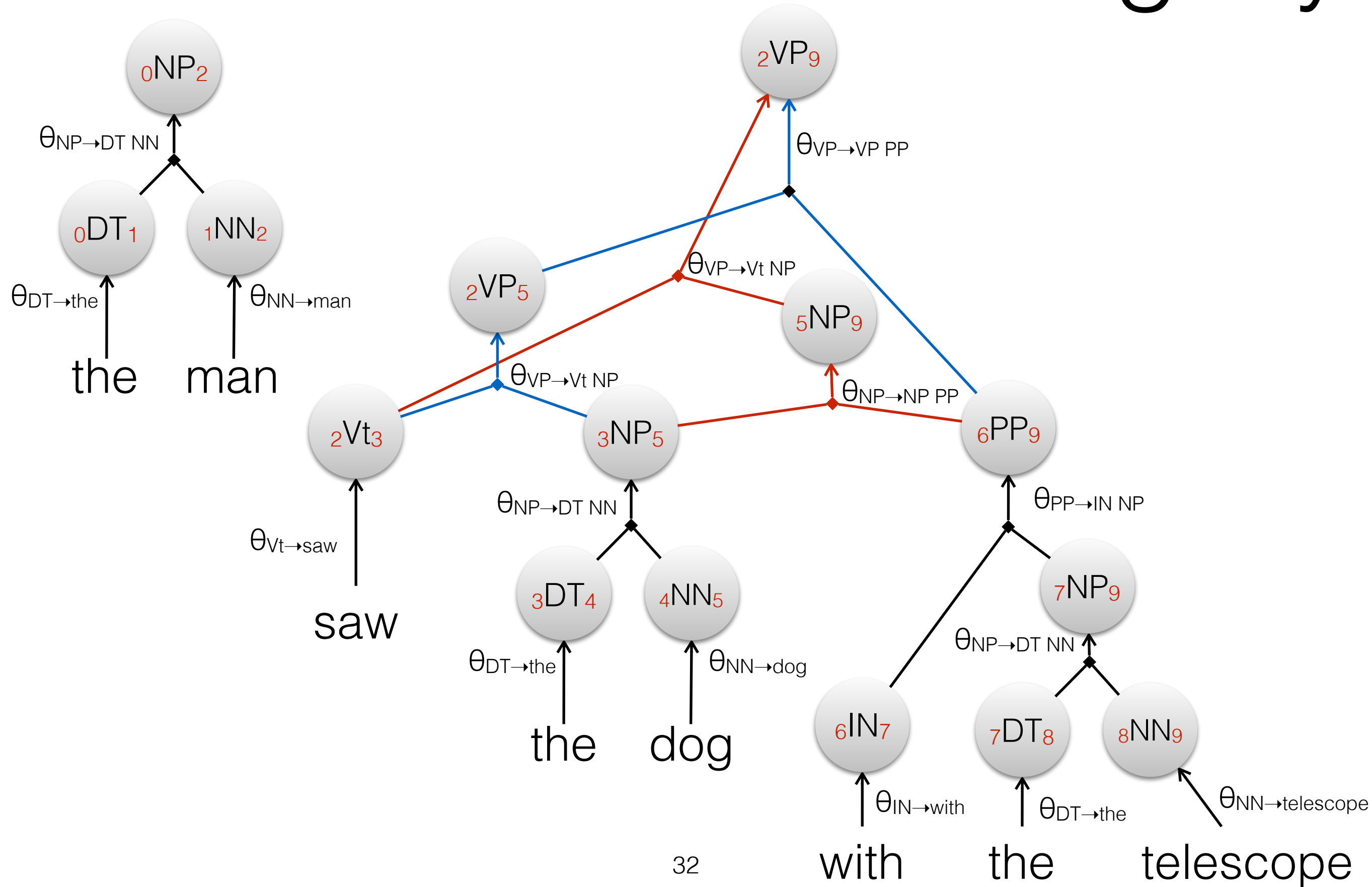
Ambiguity



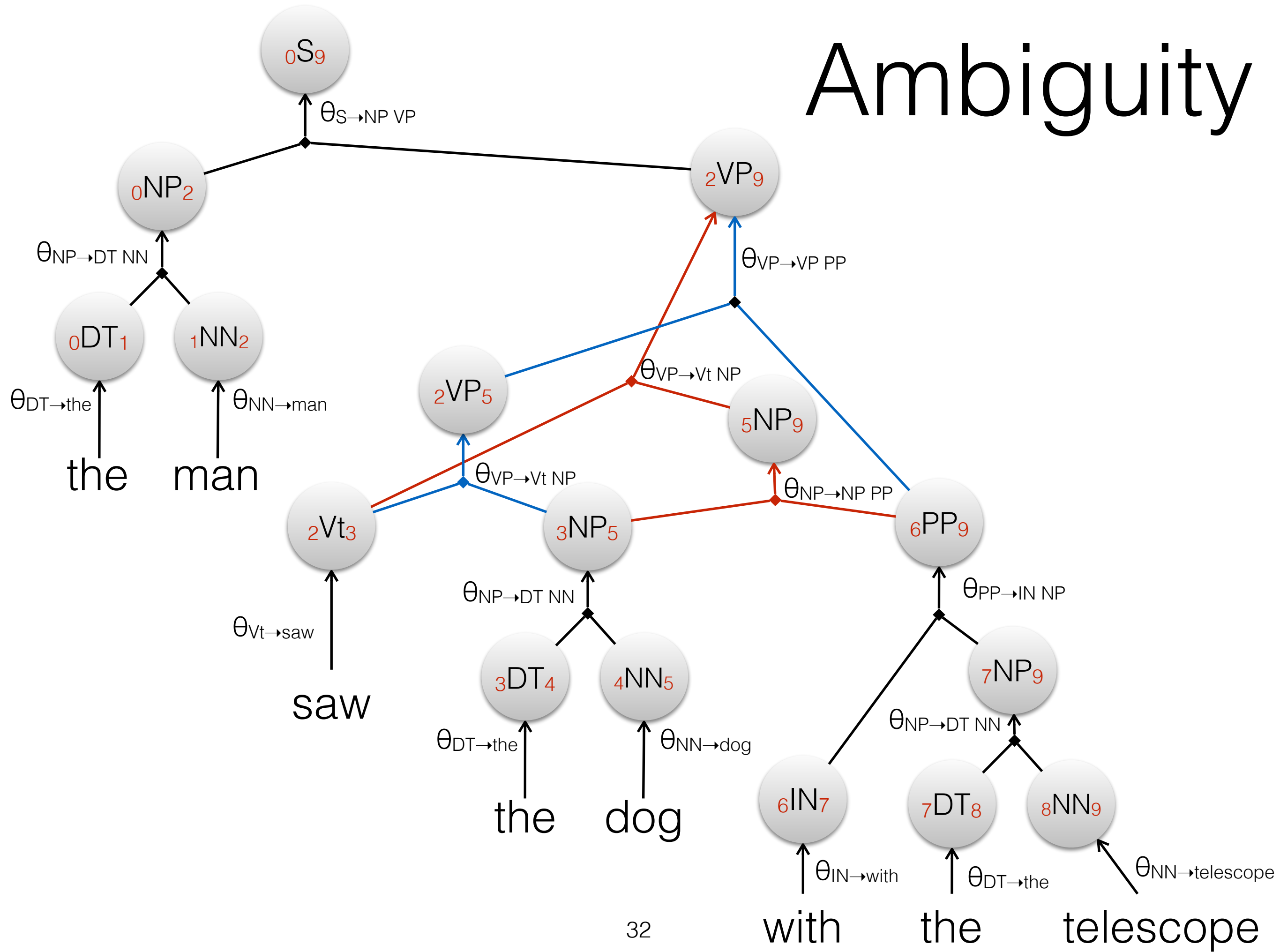
Ambiguity



Ambiguity



Ambiguity



Complexity

Complexity

Item form: $[i, X \rightarrow \alpha \blacksquare \bullet \beta \square, j]$

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- Each rule segments the input $w_1 \dots w_n$

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Every CFG can be written in CNF (max arity = 2)

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- In total we get up to 3 indices ranging from $0 \dots n$

Complexity

Item form: $[i, X \rightarrow \alpha \blacksquare \bullet \beta \square, j]$

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Every CFG can be written in CNF (max arity = 2)

- In total we get up to 3 indices ranging from $0 \dots n$
- $O(n^3)$ annotated rules

Bitext Parsing

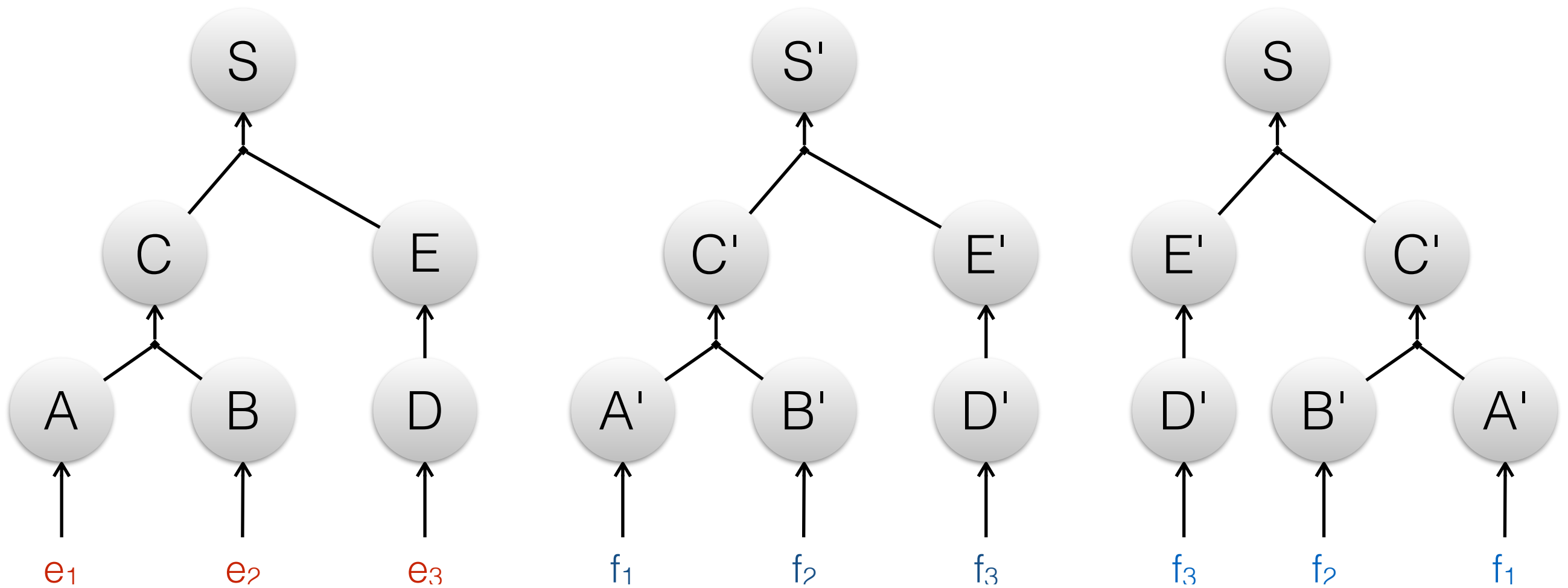
Imagine we have **two** streams of text

the man sleeps \Leftrightarrow dort l' homme

We want to parse both strings **simultaneously**
such that their trees are **isomorphic**

- same structure up to
- relabelling and permutation of siblings

Isomorphic trees



Synchronous Grammar

A CFG **paired** with another

- RHS symbols map **one-to-one**

Synchronous Grammar

A CFG **paired** with another

- RHS symbols map **one-to-one**

English French

Synchronous Grammar

A CFG **paired** with another

- RHS symbols map **one-to-one**

	English	French	
$X \rightarrow A$		A	copy

Synchronous Grammar

A CFG **paired** with another

- RHS symbols map **one-to-one**

	English	French	
$X \rightarrow$	A	A	copy
$X \rightarrow$	B C	B C	copy

Synchronous Grammar

A CFG **paired** with another

- RHS symbols map **one-to-one**

	English	French	
$X \rightarrow$	A	A	copy
$X \rightarrow$	B C	B C	copy
		C B	invert

Synchronous Grammar

A CFG **paired** with another

- RHS symbols map **one-to-one**

	English	French	
$X \rightarrow$	A	A	copy
$X \rightarrow$	B C	B C	copy
		C B	invert
$X \rightarrow$	e	f	transduce

Parse E

Parse with the English side of the grammar

${}_0S_3 \rightarrow {}_0NP_2 {}_2VP_3$

${}_0NP_2 \rightarrow {}_0DT_1 {}_1NN_2$

${}_2VP_3 \rightarrow {}_2Vi_3$

${}_0DT_1 \rightarrow \text{the}$

${}_1NN_2 \rightarrow \text{man}$

${}_2Vi_3 \rightarrow \text{sleeps}$

Projection

Projection

English

French

Projection

	English	French
${}_0S_3 \rightarrow$	${}_0NP_2 {}_2VP_3$	${}_0NP_2 {}_2VP_3$

Projection

	English	French
${}_0S_3 \rightarrow$	${}_0NP_2 {}_2VP_3$	${}_0NP_2 {}_2VP_3$
		${}_2VP_3 {}_0NP_2$

Projection

	English	French
${}_0S_3 \rightarrow$	${}_0NP_2 {}_2VP_3$	${}_0NP_2 {}_2VP_3$ ${}_2VP_3 {}_0NP_2$
${}_0NP_2 \rightarrow$	${}_0DT_1 {}_1NN_2$	${}_0DT_1 {}_1NN_2$

Projection

	English	French
${}_0S_3 \rightarrow$	${}_0NP_2 {}_2VP_3$	${}_0NP_2 {}_2VP_3$ ${}_2VP_3 {}_0NP_2$
${}_0NP_2 \rightarrow$	${}_0DT_1 {}_1NN_2$	${}_0DT_1 {}_1NN_2$ ${}_1NN_2 {}_0DT_1$

Projection

	English	French
${}_0S_3 \rightarrow$	${}_0NP_2 {}_2VP_3$	${}_0NP_2 {}_2VP_3$ ${}_2VP_3 {}_0NP_2$
${}_0NP_2 \rightarrow$	${}_0DT_1 {}_1NN_2$	${}_0DT_1 {}_1NN_2$ ${}_1NN_2 {}_0DT_1$
${}_2VP_3 \rightarrow$	${}_2Vi_3$	${}_2Vi_3$

Projection

	English	French
${}_0S_3 \rightarrow$	${}_0NP_2 {}_2VP_3$	${}_0NP_2 {}_2VP_3$ ${}_2VP_3 {}_0NP_2$
${}_0NP_2 \rightarrow$	${}_0DT_1 {}_1NN_2$	${}_0DT_1 {}_1NN_2$ ${}_1NN_2 {}_0DT_1$
${}_2VP_3 \rightarrow$	${}_2Vi_3$	${}_2Vi_3$
${}_0DT_1 \rightarrow$	the	le

Projection

	English	French
${}_0S_3 \rightarrow$	${}_0NP_2 {}_2VP_3$	${}_0NP_2 {}_2VP_3$ ${}_2VP_3 {}_0NP_2$
${}_0NP_2 \rightarrow$	${}_0DT_1 {}_1NN_2$	${}_0DT_1 {}_1NN_2$ ${}_1NN_2 {}_0DT_1$
${}_2VP_3 \rightarrow$	${}_2Vi_3$	${}_2Vi_3$
${}_0DT_1 \rightarrow$	the	le la

Projection

	English	French
${}_0S_3 \rightarrow$	${}_0NP_2 {}_2VP_3$	${}_0NP_2 {}_2VP_3$ ${}_2VP_3 {}_0NP_2$
${}_0NP_2 \rightarrow$	${}_0DT_1 {}_1NN_2$	${}_0DT_1 {}_1NN_2$ ${}_1NN_2 {}_0DT_1$
${}_2VP_3 \rightarrow$	${}_2Vi_3$	${}_2Vi_3$
${}_0DT_1 \rightarrow$	the	le la l'

Projection

	English	French
${}_0S_3 \rightarrow$	${}_0NP_2 {}_2VP_3$	${}_0NP_2 {}_2VP_3$ ${}_2VP_3 {}_0NP_2$
${}_0NP_2 \rightarrow$	${}_0DT_1 {}_1NN_2$	${}_0DT_1 {}_1NN_2$ ${}_1NN_2 {}_0DT_1$
${}_2VP_3 \rightarrow$	${}_2Vi_3$	${}_2Vi_3$
${}_0DT_1 \rightarrow$	the	le la l'
${}_1NN_2 \rightarrow$	man	homme

Projection

	English	French
${}_0S_3 \rightarrow$	${}_0NP_2 {}_2VP_3$	${}_0NP_2 {}_2VP_3$ ${}_2VP_3 {}_0NP_2$
${}_0NP_2 \rightarrow$	${}_0DT_1 {}_1NN_2$	${}_0DT_1 {}_1NN_2$ ${}_1NN_2 {}_0DT_1$
${}_2VP_3 \rightarrow$	${}_2Vi_3$	${}_2Vi_3$
${}_0DT_1 \rightarrow$	the	le la l'
${}_1NN_2 \rightarrow$	man	homme
${}_2Vi_3 \rightarrow$	sleeps	dort

French Grammar

French

${}_0S_3 \rightarrow$

${}_0NP_2 {}_2VP_3$

${}_2VP_3 {}_0NP_2$

${}_0NP_2 \rightarrow$

${}_0DT_1 {}_1NN_2$

${}_1NN_2 {}_0DT_1$

${}_2VP_3 \rightarrow$

${}_2Vi_3$

${}_0DT_1 \rightarrow$

le

la

l'

${}_1NN_2 \rightarrow$

homme

${}_2Vi_3 \rightarrow$

dort

Parse F

French

${}_0S_3 \rightarrow {}_0NP_2 {}_2VP_3$

${}_2VP_3 {}_0NP_2$

${}_0NP_2 \rightarrow {}_0DT_1 {}_1NN_2$

${}_1NN_2 {}_0DT_1$

${}_2VP_3 \rightarrow {}_2Vi_3$

${}_0DT_1 \rightarrow$ le

la

l'

${}_1NN_2 \rightarrow$ homme

${}_2Vi_3 \rightarrow$ dort

Parse F

French

${}_0S_3 \rightarrow {}_0NP_2 {}_2VP_3$

${}_2VP_3 {}_0NP_2$

${}_0NP_2 \rightarrow {}_0DT_1 {}_1NN_2$

${}_1NN_2 {}_0DT_1$

${}_2VP_3 \rightarrow {}_2Vi_3$

${}_0DT_1 \rightarrow$ le

la

l'

${}_1NN_2 \rightarrow$ homme

${}_2Vi_3 \rightarrow$ dort

${}_0$ dort ${}_1$ l' ${}_2$ homme ${}_3$

Parse F

French

${}_0S_3 \rightarrow {}_0NP_2 {}_2VP_3$

${}_2VP_3 {}_0NP_2$

${}_0NP_2 \rightarrow {}_0DT_1 {}_1NN_2$

${}_1NN_2 {}_0DT_1$

${}_2VP_3 \rightarrow {}_2Vi_3$

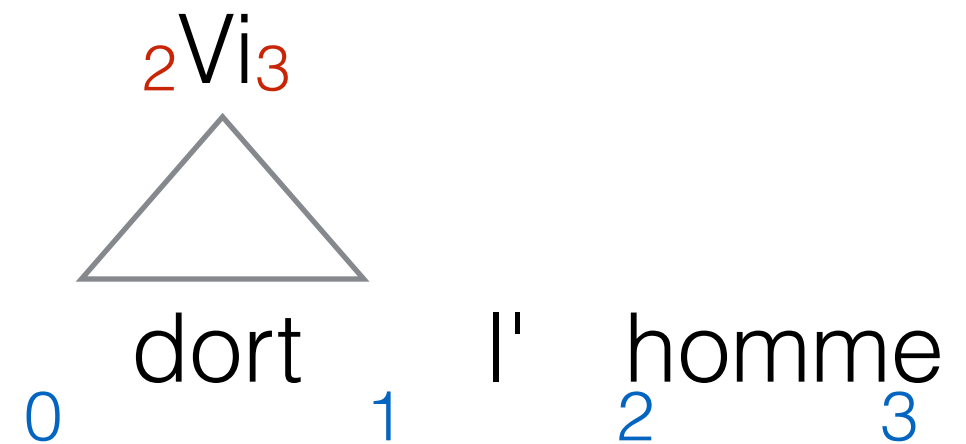
${}_0DT_1 \rightarrow$ le

la

l'

${}_1NN_2 \rightarrow$ homme

${}_2Vi_3 \rightarrow$ dort



Parse F

French

${}_0S_3 \rightarrow {}_0NP_2 {}_2VP_3$

${}_2VP_3 \rightarrow {}_2VP_3 {}_0NP_2$

${}_0NP_2 \rightarrow {}_0DT_1 {}_1NN_2$

${}_1NN_2 \rightarrow {}_1NN_2 {}_0DT_1$

${}_2VP_3 \rightarrow {}_2Vi_3$

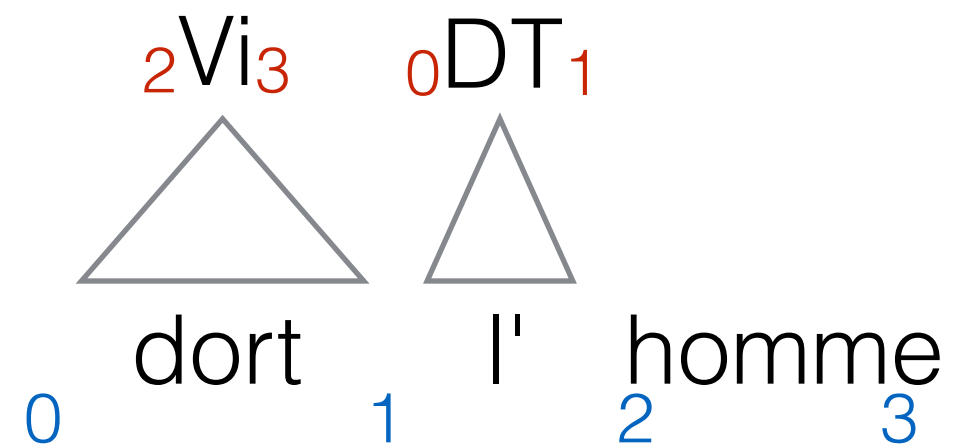
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la

l'

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Parse F

French

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${}_1NN_2 \rightarrow {}_1NN_2 {}_0DT_1$

${}_2VP_3 \rightarrow {}_2Vi_3$

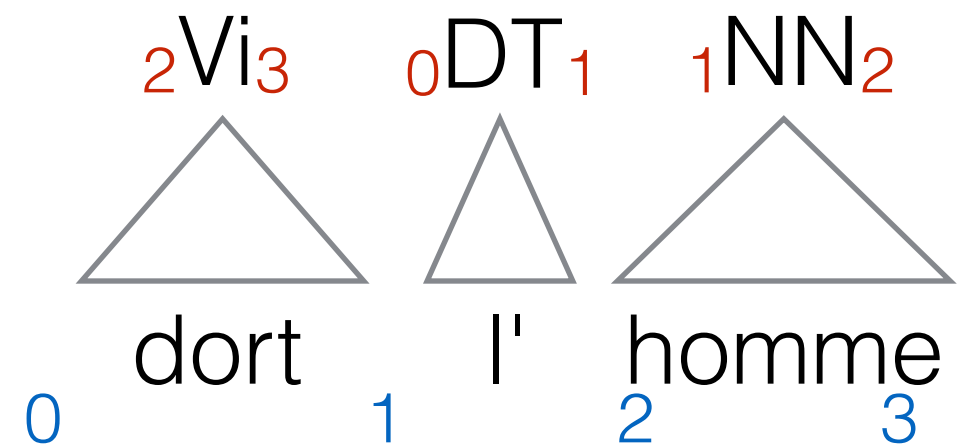
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${}_2VP_3 {}_0NP_2$

${}_0NP_2 \rightarrow {}_0DT_1 {}_1NN_2$

${}_1NN_2 {}_0DT_1$

${}_2VP_3 \rightarrow {}_2Vi_3$

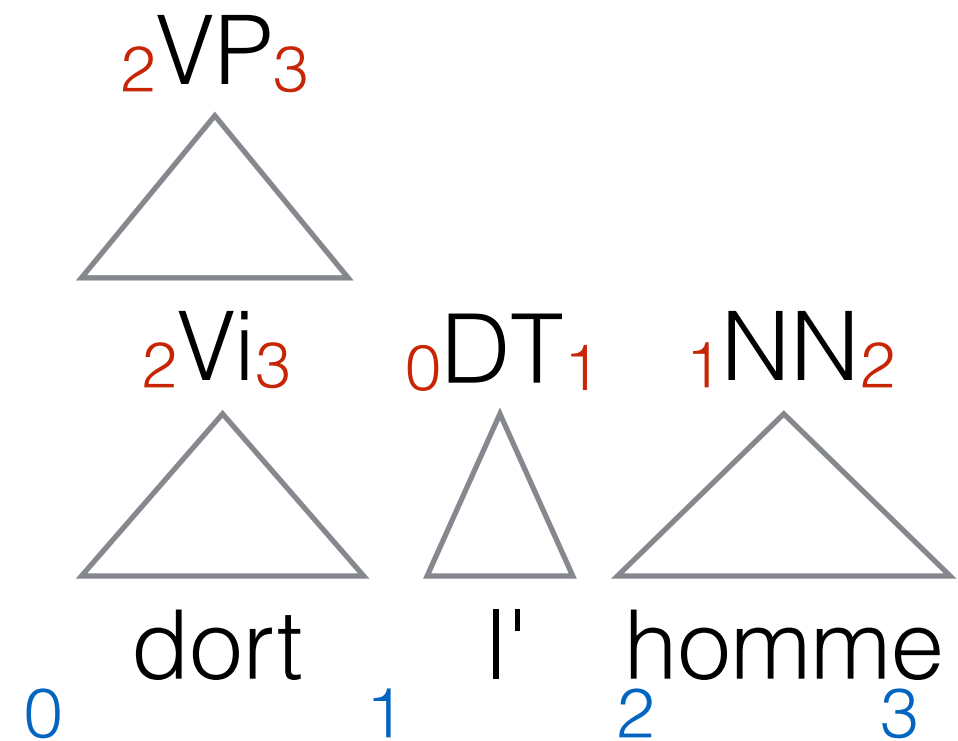
${}_0DT_1 \rightarrow$ le

la

l'

${}_1NN_2 \rightarrow$ homme

${}_2Vi_3 \rightarrow$ dort



Parse F

French

${}_0S_3 \rightarrow {}_0NP_2 {}_2VP_3$

${}_2VP_3 {}_0NP_2$

${}_0NP_2 \rightarrow {}_0DT_1 {}_1NN_2$

${}_1NN_2 {}_0DT_1$

${}_2VP_3 \rightarrow {}_2Vi_3$

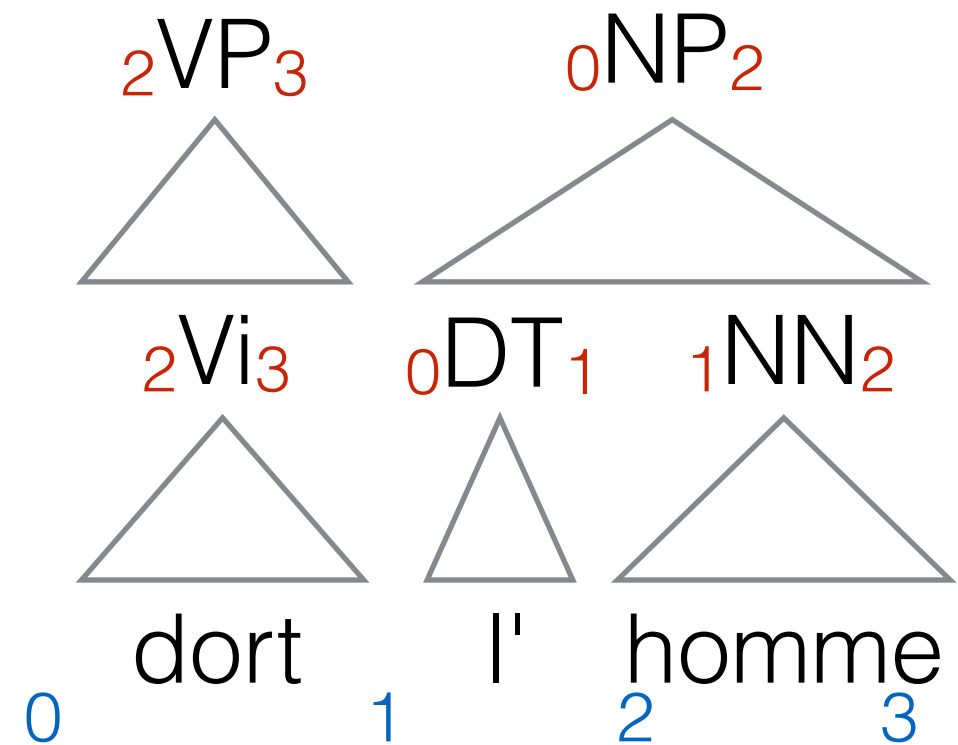
${}_0DT_1 \rightarrow$ le

la

l'

${}_1NN_2 \rightarrow$ homme

${}_2Vi_3 \rightarrow$ dort



Parse F

French

${}_0S_3 \rightarrow {}_0NP_2 {}_2VP_3$

${}_2VP_3 \rightarrow {}_2VP_3 {}_0NP_2$

${}_0NP_2 \rightarrow {}_0DT_1 {}_1NN_2$

${}_1NN_2 \rightarrow {}_1NN_2 {}_0DT_1$

${}_2VP_3 \rightarrow {}_2Vi_3$

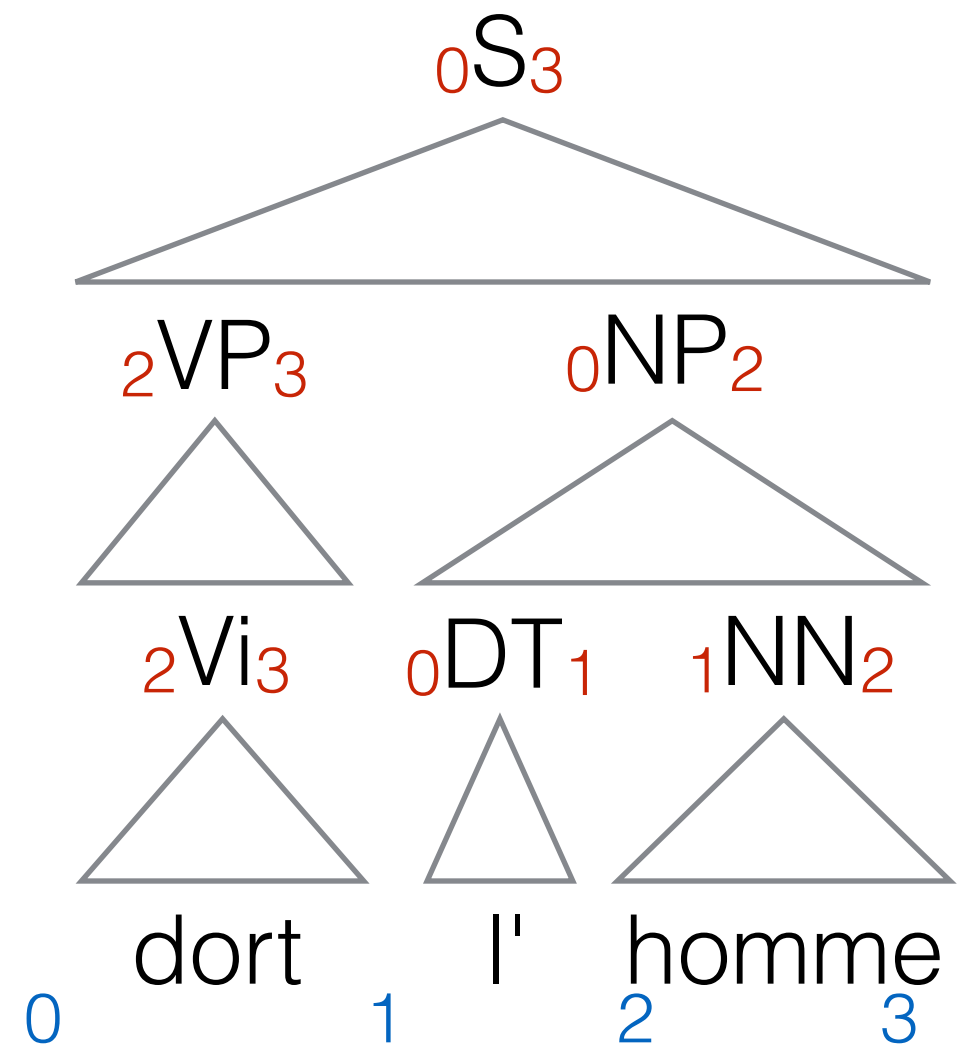
${}_0DT_1 \rightarrow$ le

la

l'

${}_1NN_2 \rightarrow$ homme

${}_2Vi_3 \rightarrow$ dort



Cascade of Monolingual Parsers

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- e.g. bitext parsing

Biproduct: alignments

French

${}_0S_3 \rightarrow {}_0NP_2 {}_2VP_3$

${}_2VP_3 {}_0NP_2$

${}_0NP_2 \rightarrow {}_0DT_1 {}_1NN_2$

${}_1NN_2 {}_0DT_1$

${}_2VP_3 \rightarrow {}_2Vi_3$

${}_0DT_1 \rightarrow$ le

la

l'

${}_1NN_2 \rightarrow$ ${}_1NN_2 \rightarrow$ homme

${}_2Vi_3 \rightarrow$ ${}_2Vi_3 \rightarrow$ dort

Biproduct: alignments

0 dort 1 l' 2 homme 3

French

${}_0S_3 \rightarrow {}_0NP_2 {}_2VP_3$

${}_2VP_3 {}_0NP_2$

${}_0NP_2 \rightarrow {}_0DT_1 {}_1NN_2$

${}_1NN_2 {}_0DT_1$

${}_2VP_3 \rightarrow {}_2Vi_3$

${}_0DT_1 \rightarrow$ le

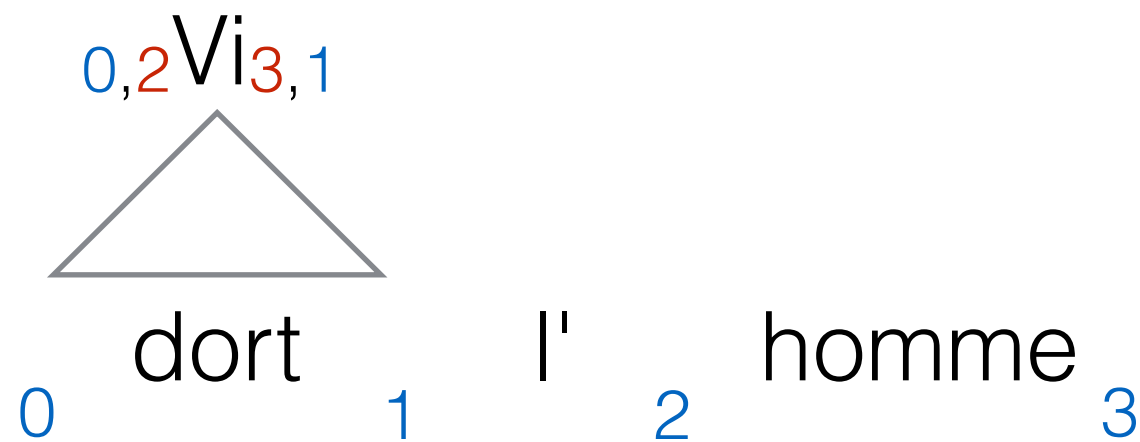
la

l'

${}_1NN_2 \rightarrow {}_1NN_2 \rightarrow$ homme

${}_2Vi_3 \rightarrow {}_2Vi_3 \rightarrow$ dort

Biproduct: alignments



French

${}^0S_3 \rightarrow {}^0NP_2 {}^2VP_3$

${}^2VP_3 {}^0NP_2$

${}^0NP_2 \rightarrow {}^0DT_1 {}^1NN_2$

${}^1NN_2 {}^0DT_1$

${}^2VP_3 \rightarrow {}^2Vi_3$

${}^0DT_1 \rightarrow$ le

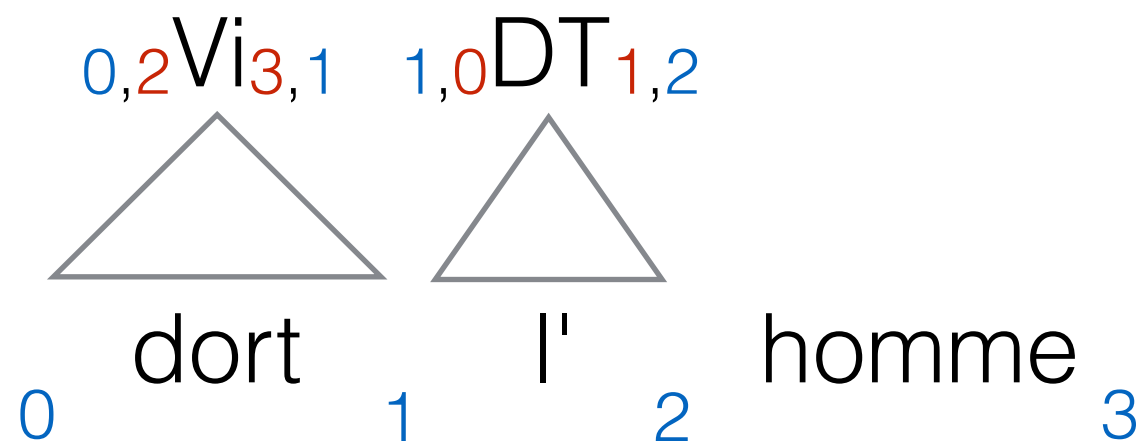
la

l'

${}^1NN_2 \rightarrow {}^1NN_2 \rightarrow$ homme

${}^2Vi_3 \rightarrow {}^2Vi_3 \rightarrow$ dort

Biproduct: alignments



French

$0S_3 \rightarrow 0NP_2 2VP_3$

$2VP_3 0NP_2$

$0NP_2 \rightarrow 0DT_1 1NN_2$

$1NN_2 0DT_1$

$2VP_3 \rightarrow 2Vi_3$

$0DT_1 \rightarrow$ le

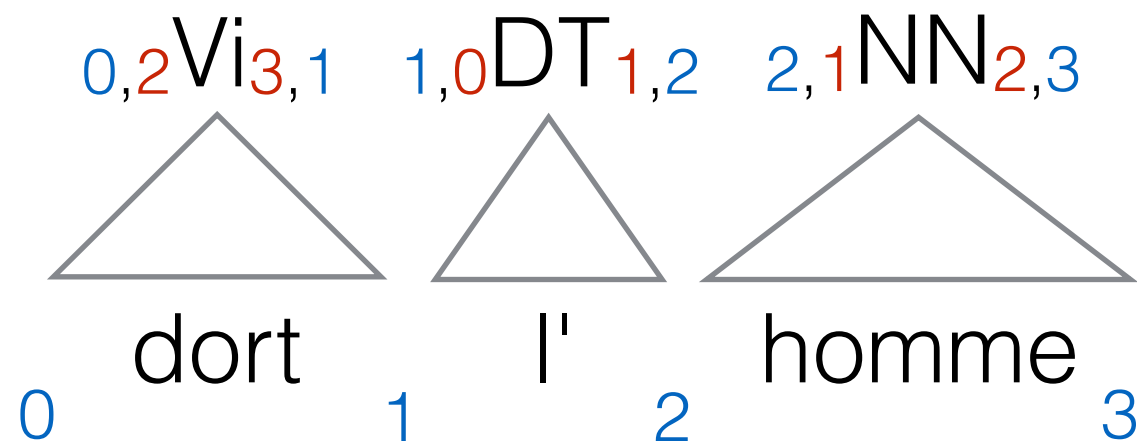
la

l'

$1NN_2 \rightarrow 1NN_2 \rightarrow$ homme

$2Vi_3 \rightarrow 2Vi_3 \rightarrow$ dort

Biproduct: alignments



French

$0S_3 \rightarrow 0NP_2 2VP_3$

$2VP_3 0NP_2$

$0NP_2 \rightarrow 0DT_1 1NN_2$

$1NN_2 0DT_1$

$2VP_3 \rightarrow 2Vi_3$

$0DT_1 \rightarrow$ le

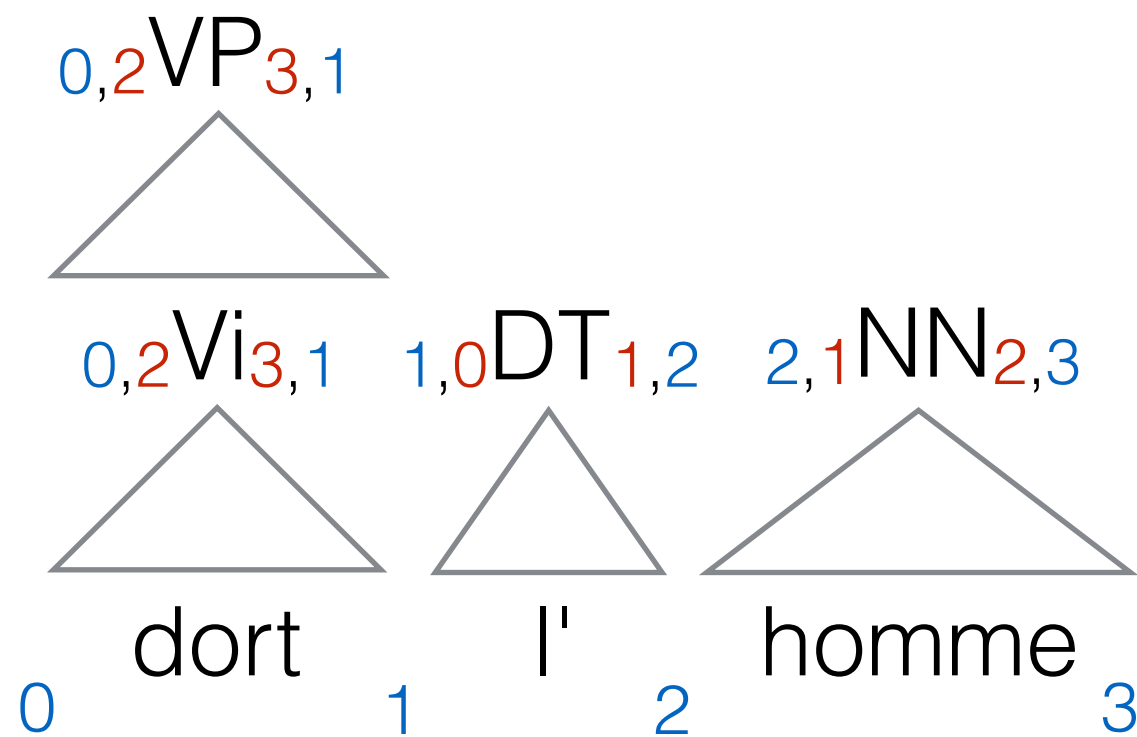
la

l'

$1NN_2 \rightarrow 1NN_2 \rightarrow$ homme

$2Vi_3 \rightarrow 2Vi_3 \rightarrow$ dort

Biproduct: alignments



French

$0S_3 \rightarrow 0NP_2 2VP_3$

$2VP_3 0NP_2$

$0NP_2 \rightarrow 0DT_1 1NN_2$

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$2VP_3 \rightarrow 2Vi_3$

$0DT_1 \rightarrow$ le

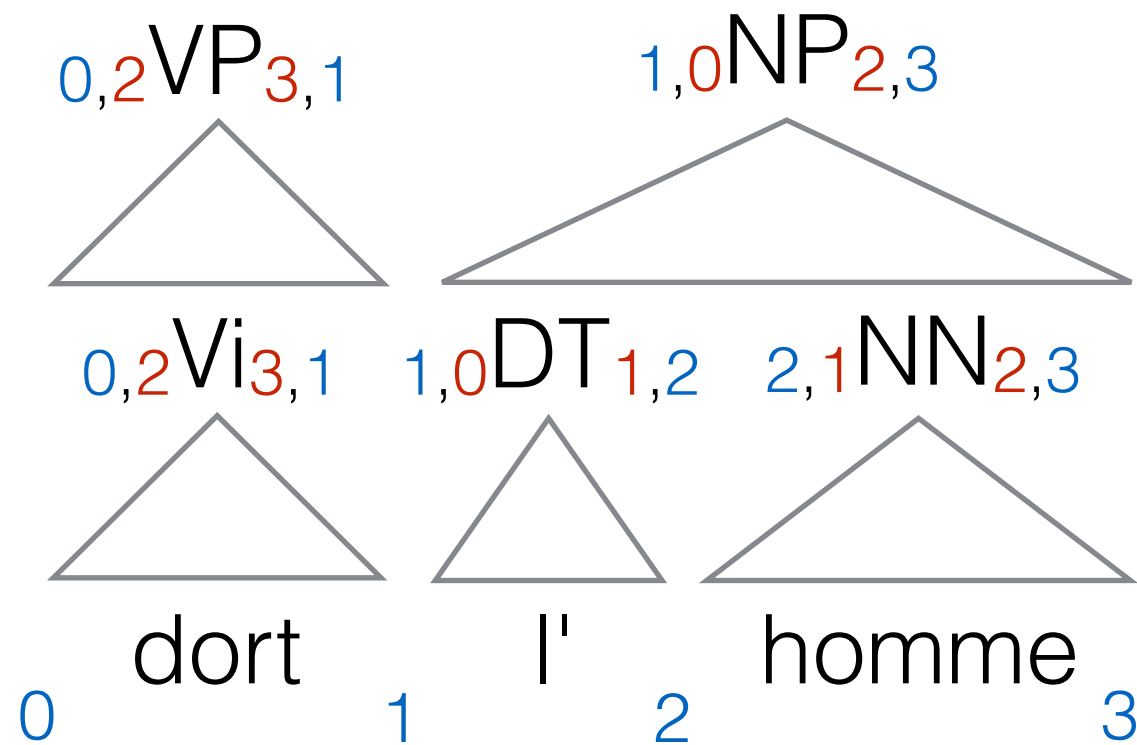
la

l'

$1NN_2 \rightarrow 1NN_2 \rightarrow$ homme

$2Vi_3 \rightarrow 2Vi_3 \rightarrow$ dort

Biproduct: alignments



French

$0S_3 \rightarrow 0NP_2 2VP_3$

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$0NP_2 \rightarrow 0DT_1 1NN_2$

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$0DT_1 \rightarrow$ le

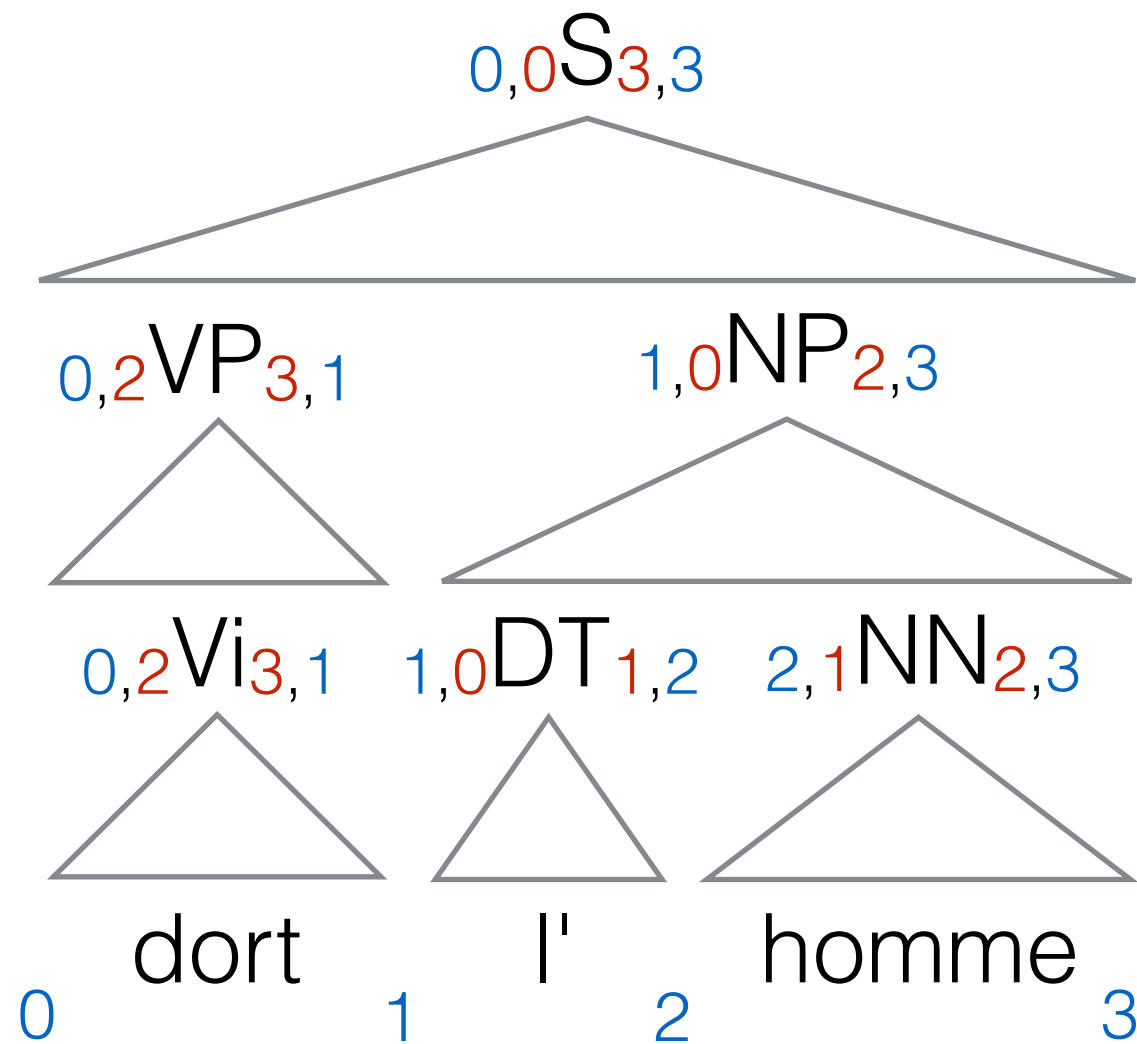
la

l'

$1NN_2 \rightarrow 1NN_2 \rightarrow$ homme

$2Vi_3 \rightarrow 2Vi_3 \rightarrow$ dort

Biproduct: alignments



French

0 S₃ → 0 NP₂ 2 VP₃

2 VP₃ 0 NP₂

0 NP₂ → 0 DT₁ 1 NN₂

1 NN₂ 0 DT₁

2 VP₃ → 2 Vi₃

0 DT₁ → le

la

l'

1 NN₂ → 1 NN₂ → homme

2 Vi₃ → 2 Vi₃ → dort

Complexity

- $O(l^3 \times m^3)$
 - where l is the length of the English string
 - and m is the length of the French string
- Joint parsing or cascade of parsers has the same theoretical complexity
- Can cascading be more efficient on average?
Why?

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